Coasian Dynamics or Failures? The Role of Trading-Up Opportunities

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Abstract

This paper develops an analytical framework that captures a broad class of monopoly pricing problems, aiming to explain why Coasian dynamics emerge in some settings while Coasian failures arise in others. We introduce the notion of trading-up opportunities and show that they are the driving force behind Coasian dynamics. In particular, pricing dynamics do not emerge in the absence of trading-up opportunities—a Coasian failure. Instead, with trading-up opportunities, pricing dynamics arise until these opportunities are exhausted or the game ends. We show how our analysis generalizes to transitional games where one variety is only indirectly accessible.

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1 Introduction

The analysis of dynamic monopoly pricing constitutes a long-standing challenge in economic theory. It is well known that Coasian dynamics are key for understanding monopoly pricing: The monopolistic seller of a single durable good who cannot commit to future prices has an incentive to lower prices over time, because high-value buyers purchase and leave the market early, whereas low-value buyers remain in the market (negative selection).[1] As a consequence, forward-looking buyers have an incentive to strategically delay their purchase and wait for lower prices. However, recent research has highlighted that Coasian dynamics fail to emerge in other settings without commitment—giving rise to so-called Coasian failures.

For example, Board and Pycia [2014] show that Coasian dynamics fail to emerge if the potential buyers of a durable good have access to an outside option with strictly positive value that ends the game. Similarly, Coasian dynamics fail to emerge if high-value rather than low-value buyers remain in the market (positive selection) for a rental good [Tirole, 2016].[2] Finally, if the seller offers two durable varieties rather than one, then, although Coasian dynamics do emerge, they do not necessarily lead to marginal cost pricing in the limit [Nava and Schiraldi, 2019]. Why is it, then, that Coasian dynamics emerge in some settings while Coasian failures arise in others?

This paper develops an analytical framework that captures a broad class of monopoly pricing problems, aiming to explain why Coasian dynamics emerge in some settings while Coasian failures arise in others, and offers a simple approach to determining whether pricing dynamics emerge in Perfect Bayesian Equilibrium (PBE).

[1] The lack of commitment constrains the monopolist's market power, and in the limit the price converges to marginal cost if all trade takes place in the "twinkle of an eye", as conjectured by Coase [1972] and formally established by Stokey [1981], Bulow [1982], Fudenberg et al. [1985], Gul et al. [1986], and Ausubel and Deneckere [1989].

[2] If the seller offers a rental good and both negative selection (for non-buyers) and positive selection (for loyal buyers) are at work, then Coasian dynamics for the prices offered to non-buyers lead to "behavior-based pricing" [Acquisti and Varian, 2005, Armstrong, 2006, Fudenberg and Villas-Boas, 2007, Buehler and Eschenbaum, 2020].

To fix ideas, consider the following example based on Hart and Tirole [1988]. There is a single risk-neutral buyer whose private (per-period) valuation for a single product is v^H with probability f and $v^L \leq v^H$ with probability $1-f$. The outside option has a valuation of zero. In the static game, a single supplier sets the monopoly price

$$p^m = \begin{cases} v^H, \text{ if } fv^H > v^L; \\ v^L, \text{ if } fv^H \leq v^L. \end{cases}$$

Therefore, if $fv^H \leq v^L$, the seller sets the price such that both types buy (market clearing) their most-preferred option (efficiency). In addition, we will argue that there are no trading-up opportunities because following the purchase no buyer type has access to a higher-valued option.[3] If $fv^H > v^L$ instead, only the high type v^H buys, leaving a trading-up opportunity for the seller who could also sell to the low type v^L.

In a two-period version of the game where the product is durable and the discount factor is $\delta \in [0,1)$, type $i \in \{H, L\}$ buys in the first period if and only if $(1+\delta)v^i - p^1 \geq \delta(v^i - p^2) \Leftrightarrow v^i \geq p^1 - \delta p^2$, where p^t is the price in period t. If the seller sets prices to separate buyer types, the profit-maximizing strategy is to set $p^2 = v^L$ and $p^1 = v^H + \delta v^L$, which results in a profit of $fv^H + \delta v^L$.[4] If the seller pools both types together, both types buy (market clearing) their most-preferred option (efficiency) at the profit-maximizing prices $p_1 = p_2 = (1+\delta)v^L$, which result in a profit of $(1+\delta)v^L$ and exhaust all trading-up opportunities.[5] Thus, the seller separates types, thereby leaving a trading-up opportunity in the repeated game, if and only if $fv^H > v^L$.

This example illustrates that, whenever setting the price to v^L is optimal in the static game, it is also optimal to set the same constant price in the repeated version. That is, if the static monopoly allocation leaves no trading-up opportunities, then pricing dynamics do not emerge in the repeated game.

[3]As will become clear below, trading-up opportunities (rather than market-clearing or efficiency conditions) drive the pricing dynamics when there are multiple varieties. We will formalize the notion of trading-up opportunities in Section 2.2.

[4]The high type has valuation $(1+\delta)v^H$ and thus obtains an information rent.

[5]All prices $p^2 \geq v^L$ are profit maximizing and resulting in the same allocation.

Our paper applies this idea to the analysis of a broad class of dynamic monopoly pricing problems that allows for multiple durable, multiple rental, or a mix of varieties. We formalize the notion of trading-up opportunities and show that it is the existence of trading-up opportunities (where some buyer types have access to a higher-valued option) that drives the pricing dynamics. Coasian failures, in turn, arise in the absence of trading-up opportunities. We note that, in settings with multiple durable varieties, the absence of trading-up opportunities is equivalent to market clearing [Nava and Schiraldi, 2019]. Instead, with multiple rental varieties, the absence of trading-up opportunities requires efficiency because all types must buy their most-preferred variety. Finally, with mixed varieties, the absence of trading-up opportunities requires that the rental variety is the most-preferred one for all types that buy it.

We study a setup with a single seller with constant marginal cost normalized to zero. The seller chooses prices for two varieties of a good, facing a single buyer who is privately informed about her valuations for these varieties. We assume that the seller cannot commit to future prices. In each period, the buyer selects one of three states: she either purchases one of the two varieties or chooses the outside option (no consumption). A fixed set of admissible transitions between these three states governs the choices that are available to the buyer. In particular, if one of the two varieties is not accessible to the buyer throughout the game, our setting reduces to a one-variety problem. If the buyer cannot select the outside option in a given period and must stick with her previous consumption choice, we impose a price of zero for her previous consumption choice to prevent expropriation. It is convenient to think of an absorbing variety as a durable good that can be sold once and for all future periods, whereas a variety that can be purchased in every period separately can be viewed as a rental good. We are interested in characterizing the pricing dynamics in PBE.

We derive three key results. *First*, we show that the seller can do no better than charge static monopoly prices if there are no trading-up opportunities in the static monopoly outcome, regardless of whether durable, rental, or mixed varieties are offered. Intuitively, the result follows because there are no buyer types that can benefit from switching to another consumption path if there are no trading-up opportunities in the static monopoly outcome. The result implies that Coasian dynamics do not emerge in settings without trading-up opportunities in the static monopoly outcome, irrespective of the seller's commitment ability. The result is reminiscent of Tirole [2016]'s finding that it is optimal to charge the static monopoly price to loyal buyers of a single rental good in a positive selection setting, where trading-up opportunities are eliminated by an absorbing outside option. It is also related to Board and Pycia [2014]'s analysis, where trading-up opportunities in the static monopoly outcome are eliminated by an additional outside option with a strictly positive valuation. The key difference is that, in our setting, the seller can set the price of the additional purchase option (the second variety) endogenously. *Second*, we show that, after any history at which there are trading-up opportunities, the seller lowers prices until all trading-up opportunities are exhausted or the game ends. Yet, dynamic prices do not fall below the prices $\bar{p}$ associated with the seller-optimal outcome in the static game that leaves no trading-up opportunities. In addition, the seller's present discounted profit is bounded from below by the repeated static profit $\pi(\bar{p})$ that leaves no trading-up opportunities, which implies that the seller can obtain a positive profit in various settings.[6] *Third*, we show that our analytical approach also applies to transitional games, where one of the varieties is only indirectly available from the initial state (via the other variety), provided that the associated static games are properly defined.

Our analysis highlights that, for a broad class of dynamic monopoly pricing problems, the pricing dynamics depend on whether the monopoly outcome in the (properly defined) static game leaves trading-up opportunities to the seller. If the monopoly outcome in the static game leaves no trading-up opportunities, then the seller does not face a commitment problem and can implement the repeated static monopoly outcome. Instead, if there are trading-up opportunities in the static monopoly outcome, then the seller lowers prices to trade up the buyer to higher-

[6]In the previous example with one durable product, we have $\bar{p} = v^L$ and profit $\pi(\bar{p}) = (1 + \delta)v^L$.

valued options, and a zero-profit lower bound applies in some settings, but not in general. Our analysis suggests that the essence of Coase's insight generalizes to the pricing of multiple non-durable varieties: pricing dynamics emerge whenever some buyer types have access to a higher-valued consumption option.

This paper adds to an extensive literature on the pricing of a single durable good [e.g Coase, 1972, Fudenberg et al., 1985, Hart and Tirole, 1988, Sobel, 1991, Kahn, 1986, Bond and Samuelson, 1984, Fuchs and Skrzypacz, 2010], of multiple varieties of a durable good [e.g., Nava and Schiraldi, 2019, Board and Pycia, 2014], and of vertically differentiated durable products [Hahn, 2006, Inderst, 2008, Takeyama, 2002]. Our work differs by proposing a unified analytical framework that also captures novel settings with two rental or mixed varieties and showing that trading-up opportunities drive the pricing dynamics in all of these settings. We further contribute to the analysis of positive selection [Tirole, 2016] by showing how it can be extended to multiple varieties. In addition, we add to the literature on behavior-based pricing [e.g. Acquisti and Varian, 2005, Armstrong, 2006, Fudenberg and Villas-Boas, 2007, Taylor, 2004, Buehler and Eschenbaum, 2020] by considering multiple varieties. In line with the bulk of previous research, we focus on price posting and abstract from smart contracts [Brzustowski et al., 2023] or bundling [Rochet and Thanassoulis, 2019]. Finally, we note that our notion of "trading up" is different from "upselling" [e.g., Blattberg et al., 2008, Aydin and Ziya, 2008, Wilkie et al., 1998] and offering add-ons [Yu et al., 2025] because it applies to buyers *and* non-buyers.

The remainder of the paper is organized as follows. Section 2 introduces the setup, formalizes the notion of trading-up opportunities, and discusses various nested cases. Section 3 derives a simple skimming property for the unified analytical framework and characterizes dynamic monopoly pricing with and without trading-up opportunities in static monopoly. We illustrate our results with two examples in Section 4. Section 5 extends the analysis to transitional games. Section 6 concludes and offers directions for future research.

2 Setup

A monopolist offers two varieties of a good, a and b, over discrete time $t = 1, ..., T$, with $T < \infty$, to a single (risk-neutral) buyer with unit demand for the good in every period. Following Nava and Schiraldi [2019], the buyer's value profile $v = (v_a, v_b)$ is fixed, private information, and distributed according to a probability measure $\mathcal{F}$ defined on the unit square $[0, 1]^2$. V is the support (i.e., the smallest closed set such that $\mathcal{F}([0, 1]^2 \setminus V) = 0$). Let $\mathcal{B}(V)$ be the Borel sigma-algebra of V. We assume that the measure $\mathcal{F}$ is atomless: for any $E \in \mathcal{B}(V)$ with $\mathcal{F}(E) > 0$, there exists $E' \in \mathcal{B}(V)$ with $E' \subset E$ and $0 < \mathcal{F}(E') < \mathcal{F}(E)$. Moreover, let the support V be convex. The value of the outside option is normalized to zero, and players share the same discount factor $\delta \in [0, 1)$.[7]

In every period $t \geq 1$, the buyer makes a discrete choice $x^t \in X$, where

$$X \equiv \{a, b, o\}$$

is the set of states, with varieties $a = (1, 0)$ and $b = (0, 1)$, and the outside option $o = (0, 0)$. Let $x^0 \in X$ denote the initial state. A sequence of choices x^t from period t onward traces out a *consumption path* $\mathbf{x}^t = (x^t, x^{t+1}, ..., x^T)$ that generates (discounted to period t) *total consumption* $\chi(\mathbf{x}^t) = \sum_{\tau=t}^{T} \delta^{\tau-t} x^\tau$. A consumption path is *admissible* if all transitions along the entire path are within the set of admissible transitions $\Gamma \subset X \times X$, where Γ is exogenous and determines how the buyer can switch between states from one period to the next. Throughout, we assume that transitions from any given state to itself are always admissible, that is, $(o, o), (a, a), (b, b) \in \Gamma$. In the main part of our analysis, we focus on settings in which each state is either directly accessible from the initial state or not accessible at all. We will consider the extension to settings where one variety is only indirectly accessible via another state ("transitional games") in Section 5.[8]

[7] We will work with the natural interpretation of the model that there is a single buyer whose type is unknown. Under the alternative interpretation, the seller faces a continuum of infinitesimal buyers who are indistinguishable to the seller and observes only the measures of sets of buyers who accept or reject [Fudenberg and Tirole, 1993, p. 400].

[8] For example, the buyer may be able to purchase a particular used car only after having leased it.

A state $x \in X$ is *absorbing* if no other state $x' \neq x$ is accessible from x, that is, if $(x, x') \notin \Gamma$. Let $\mathbf{X}(h^t)$ be the set of admissible consumption paths after history h^t. In addition, let $\Delta^t = \sum_{\tau=t}^{T} \delta^{\tau-t}$ denote the (discounted to period t) *number of periods* from t onward. For simplicity, we will omit the time superscript of Δ^t if $t = 1$ from now on.

We call a variety "durable" if it is absorbing and can only be sold once and for all future periods. A variety that allows for transitions to other states, in turn, is called a "rental" variety. To simplify exposition, we will henceforth refer to the setting with two absorbing varieties and non-absorbing initial state $x^0 = o$ as the "two durables" setting. Similarly, we will refer to the setting with all transitions being admissible and initial state $x^0 = o$ as the "two rentals" setting. Lastly, we will refer to the setting with one rental and one durable variety with initial non-absorbing state $x^0 = o$ as the "mixed varieties" setting.

Figure 1 illustrates two different settings: two rentals (panel (a)), and mixed varieties (panel (b)). The vertices indicate states $X = \{a, b, o\}$, with initial state $x^0 = o$, and the arcs and brackets $(x, x') \in \Gamma$ represent admissible transitions. Panel (a) shows that all transitions are admissible in the two rentals setting, that is, $\Gamma = \{(a, a), (b, b), (o, o), (a, b), (b, a), (o, a), (o, b), (a, o), (b, o)\}$. Panel (b) indicates that in a mixed varieties setting where b is the durable variety, the transitions $(b, o), (b, a) \notin \Gamma$ are not admissible because durable varieties are absorbing states.

2.1 Prices, Histories, and Solution Concept

In every period t, the monopolist selects a price profile $p^t = (p_a^t, p_b^t) \in [\psi, 1]^2$, with $\psi < 0$,[9] conditional on seller history h^t. The buyer then makes consumption choice $x^t \in X(h^t)$, where $X(h^t) \subseteq X$ is the set of states accessible after history h^t. A sequence of seller histories is given by $h^1 = \{x^0\}$ and $h^t = \{h^{t-1}, p^{t-1}, x^{t-1}\}$ for $t \geq 2$, where h^{t-1} is a subhistory of h^t. Similarly, a buyer history is given by $\hat{h}^t = \{h^t, p^t\}$ for $t \geq 1$.

The set of period-t seller histories is denoted by H^t, and the set of all seller histories by $H = \cup_{t=1}^{T} H^t$. Similarly, the set of period-t buyer histories is denoted

[9] The assumption on the set of prices ensures that the monopolist's action set is compact.

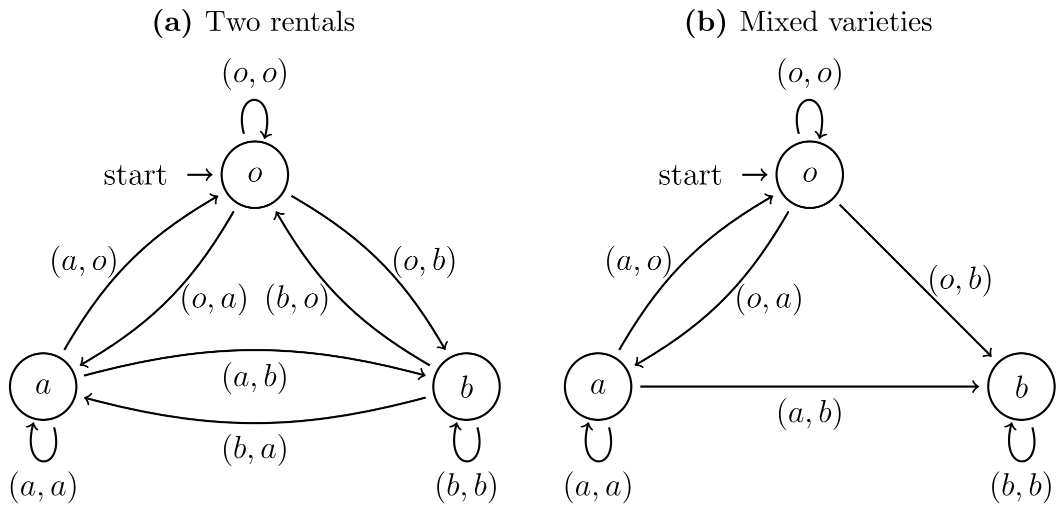


Figure 1: States and transitions in two different settings

by $\hat{H}^t$, and the set of all buyer histories by $\hat{H} = \cup_{t=1}^{T} \hat{H}^t$.

A behavioral strategy for the buyer is denoted by $\hat{\sigma}$ and determines the probability distributions over the consumption choices $x^t \in X$ made at every buyer history $\hat{h}^t$. Formally, $\hat{\sigma} : \hat{H} \times V \to s(X)$, where $s(.)$ denotes the respective simplex. In line with the literature, we assume that, at any possible history, the set of buyer types making the same consumption choice is measurable. A behavioral strategy for the seller is denoted by σ and determines the probability distribution over the prices $p^t \in [\psi, 1]^2$ set by the seller after every seller history h^t. Formally, $\sigma : H \to s([\psi, 1]^2)$.

A *Perfect Bayesian Equilibrium* (PBE) is a strategy profile $\{\sigma, \hat{\sigma}\}$ and updated beliefs about the buyer's value profile along the various consumption paths, such that actions are optimal given beliefs, and beliefs are derived from actions from Bayes' rule whenever possible.

We partition the buyer type space V depending on the history. Let $V(h^1) = V$ and

$$V(h^{t+1}) = \left\{ v \in V(h^t) \middle| x^t \in \arg\max_{x \in X(h^t)} (v - p^t) \cdot x + \delta U(v, x, \hat{h}^t) \right\}, \text{ for } t \geq 1,$$

where x^t is the last element of seller history $h^{t+1} = \{h^t, p^t, x^t\}$, p^t is the price profile selected by the seller after seller subhistory h^t, and $U(v, x, \hat{h}^t)$ is the buyer's continuation value after buyer history $\hat{h}^t = \{h^t, p^t\}$. Thus, we obtain $\cup_{h^t \in H^t} V(h^t) = V$ at any time t (i.e., no types are left behind). Accordingly, $\mathcal{F}(V(h^t))$ measures all buyer types with history h^t (i.e., all buyer types choosing x^{t-1}, facing price profile p^{t-1} after history h^{t-1}).

Importantly, we assume that if the buyer cannot transition to the outside option from variety $x^{t-1} \in \{a, b\}$ after history h^t, then the period-t price for variety x^{t-1} at this history is zero, $p^t_{x^{t-1}}(h^t) = 0$. This assumption is consistent with the natural interpretation of an absorbing variety as a durable good and excludes the expropriation of a "captured" buyer. Let $\rho(\mathbf{p}^t, \mathbf{x}^t) = \sum_{\tau=t}^{T} \delta^\tau (p^\tau \cdot x^\tau)$ denote the (discounted to period t) *total payment* made along consumption path $\mathbf{x}^t$ with price path $\mathbf{p}^t = (p^t, p^{t+1}, ..., p^T)$ after history h^t. Similarly, let $\nu(v, \mathbf{x}^t) = v \cdot \chi(\mathbf{x}^t)$ be the (discounted to period t) *total value* obtained by a buyer with value profile v along consumption path $\mathbf{x}^t$ after history h^t. We can then write the (present discounted) *net value* obtained by a buyer with value profile v along consumption path $\mathbf{x} = \mathbf{x}^1$ and the path of price profiles $\mathbf{p} = \mathbf{p}^1$ compactly as $\nu(v, \mathbf{x}) - \rho(\mathbf{p}, \mathbf{x})$.

2.2 Trading-up opportunities

We introduce the following definition.

Definition 1 (**Trading-up opportunity**). *The seller has a trading-up opportunity at history $h^t = \{h^{t-1}, p^{t-1}, x^{t-1}\}$ if there is a positive measure of buyer types who can transition to a strictly higher-valued state,*

$$\mathcal{F}\left(\left\{v \in V(h^t) \;\middle|\; x^{t-1} \notin \arg\max_{x \in X(h^t)} v \cdot x\right\}\right) > 0.$$

Definition 1 formalizes the notion that, for a trading-up opportunity to exist for the seller at history h^t, some buyer types must have access to a higher-valued consumption option than the current one.

Our notion of trading-up opportunities relates to market clearing and efficiency: In the durable case, there are no trading-up opportunities if and only if the market clears, that is, $x \neq o$ for all buyer types. In the rental case, there are no trading-up opportunities if and only if the allocation is efficient, that is, each buyer type chooses its most preferred variety $\arg\max_{x \in X(h^1)} v \cdot x$. More generally, no trading-up opportunities imply market clearing; efficiency implies no trading-up opportunities. Thus, our notion of trading-up opportunities is more restrictive than market-clearing, yet less restrictive than efficiency.

For later reference, we let Ω denote the set of price profiles $p = (p_a, p_b)$ that induce an allocation which leaves no trading-up opportunities for the seller in the static game,

$$\Omega = \left\{ p \in [\psi, 1]^2 \middle| \mathcal{F}\left(\left\{ v \in V(\{x^0, p, x^1\}) \middle| x^1 \notin \arg\max_{x \in X(h^2)} v \cdot x \right\} \right) = 0 \right\}.$$

Intuitively, any price profile $p \in \Omega$ must induce an allocation in the static game where the buyer selects either an absorbing state or the most-preferred state among those that are accessible from the initial state. Thus, in a setting with two durables, any price profile $p \in \Omega$ must implement market-clearing.[10] In a setting with two rentals, in turn, any price profile $p \in \Omega$ must implement market-clearing and efficiency.[11]

Because $\mathcal{F}$ is atomless, the static demand for variety $i \neq j \in \{a, b\}$ satisfies $d_i(p) = \mathcal{F}(v \in V | v_i - p_i \geq \max\{v_j - p_j, 0\})$, resulting in the static profit $\pi(p) = d_a(p)p_a + d_b(p)p_b$. We let $p^m \in \arg\max \pi(p)$ denote the static profit-maximizing price profile.

Finally, we let $\bar{p} \in \Omega$ denote a price profile that is associated with the maximum of the profit obtainable in the static game conditional on leaving no trading-up

[10] Following Nava and Schiraldi [2019], the set of market-clearing prices is

$$\left\{ p \in \mathbb{R}^2 \middle| \max_{i \in \{a,b\}} \{v_i - p_i\} \geq 0, \forall v \in V \right\} \supseteq \Omega.$$

[11] We follow Nava and Schiraldi [2019] in referring to price profiles which ensure that all buyers choose their preferred (accessible) variety as *efficient* because they maximize total welfare when marginal costs are normalized to zero, formally

$$\left\{ p \in \mathbb{R}^2 \middle| \arg\max_{x \in X(h^1)} (v - p) \cdot x \subseteq \arg\max_{x \in X(h^1)} v \cdot x, \forall v \in V \right\} \subseteq \Omega.$$

opportunities, $\pi(\bar{p}) = \max \pi(p)$ s.t. $p \in \Omega$. This profit maximum exists provided that $\Omega \neq \emptyset$, which is guaranteed since $p = (0, 0) \in \Omega$ (we consider the extension to transitional games in section 5).

Figure 2: Demand segments in the static game for given p (panel (a)), and profiles $p \in \Omega$ with full support (panel (b)) or linear support (panel (c)) for two rentals

(a) Static demand **(b)** $p \in \Omega$, full support **(c)** $p \in \Omega$, linear support

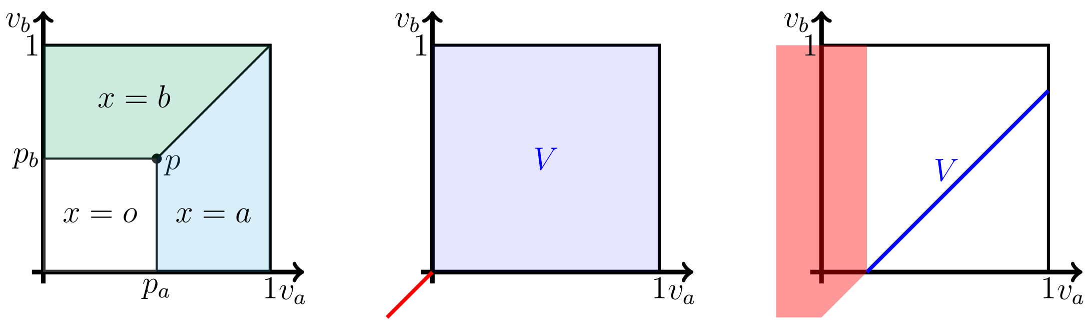


Figure 2 illustrates for the two rentals setting how buyer types self-select in the static game for a given price profile p, and depicts price profiles (in red) that satisfy $p \in \Omega$ for two different supports. Specifically, panel (a) shows the static demand segments for the price profile $p = (0.5, 0.5)$ and indicates, for instance, that all consumer types with values $v_i < p_i, \forall i$, choose the outside option, $x = o$. Panels (b) and (c) depict price profiles that leave no trading-up opportunities with full and linear support, respectively. For two rentals, $p \in \Omega$ requires that all buyer types choose their most-preferred variety, as otherwise there are trading-up opportunities from one variety to the other, or from the initial state to each variety. Thus, with full support only non-positive price profiles on the diagonal satisfy $p \in \Omega$ (panel (b)), whereas with an increasing linear support that lies to the right of the diagonal through the type space (panel (c)), any price profile that ensures $x = a$ for all types in the support satisfies $p \in \Omega$, since all buyer types prefer a to b ("vertical differentiation").

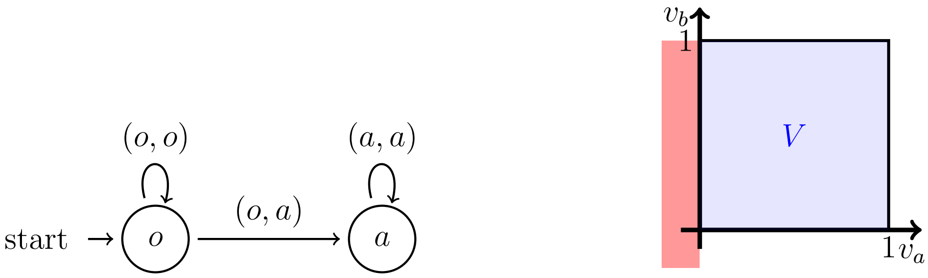
2.3 Nested cases

Our setup covers a broad class of dynamic monopoly pricing settings that can be characterized by the tuple $(x^0, \Gamma, \mathcal{F})$. It is convenient to illustrate these settings in two complementary graphs: one showing the accessible states and admissible transitions, and one showing the support V of the value profiles and the price profiles $p \in \Omega$ that leave no trading-up opportunities, respectively. Figure 3 provides three examples that are drawn for a full support V on the unit square $[0, 1]^2$.

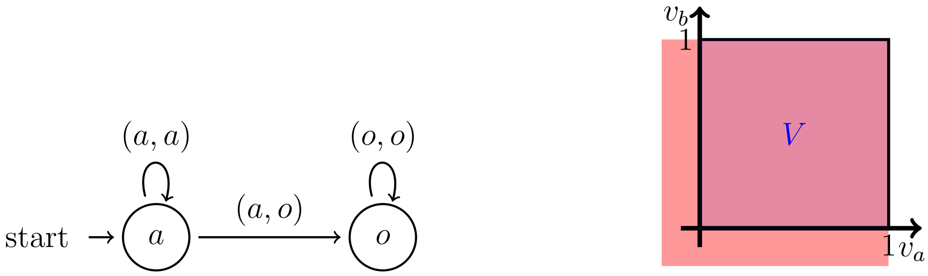
In a setting with a single durable variety a and full support (Figure 3a), $p \in \Omega$ requires that $p_a \leq 0$ (whereas p_b remains unrestricted), which implies that $\pi(\bar{p}) = 0$. In a positive selection setting with a single variety a and initial state $x^0 = a$ (Figure 3b), in turn, all price profiles satisfy $p \in \Omega$, which implies that $\pi(\bar{p}) = \pi(p^m) > 0$ with full support. Finally, in a mixed setting with rental variety a, durable variety b, and initial state $x^0 = o$ (Figure 3c), $p \in \Omega$ requires that the price of the durable variety b is non-positive, whereas the price of the rental variety a can be positive and thus $\pi(\bar{p}) > 0$ with full support.

Figure 3: Three examples: Accessible states and admissible transitions (left), and price profiles $p \in \Omega$ with a full support (right)

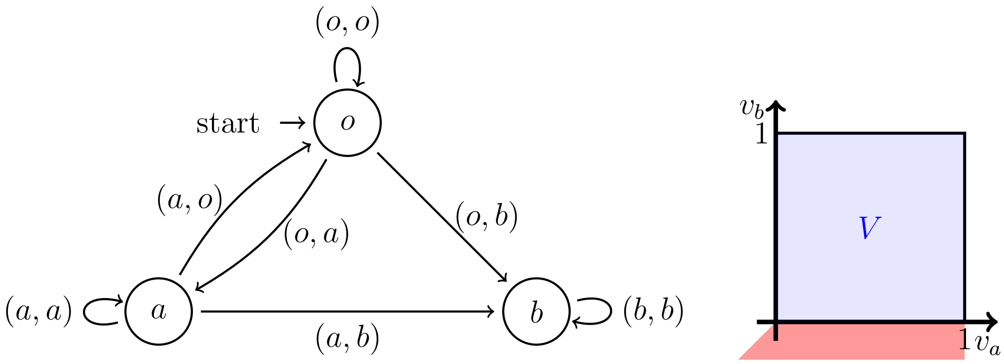
(a) Single durable variety a, with initial state $x^0 = o$



(b) Positive selection, with single variety a and initial state $x^0 = a$



(c) Mixed setting, with rental variety a, durable variety b, and initial state $x^0 = o$



3 Analysis

We now characterize dynamic monopoly pricing within the framework introduced above. We proceed in three steps. First, we provide a simple skimming result. Second, we introduce a convenient way of representing the seller's profit. Finally, we analyze optimal pricing with and without trading-up opportunities.

3.1 Skimming

We first show that the value profiles of buyer types who make the same consumption choice satisfy an intuitive sorting condition (all proofs are relegated to the Appendix).

Lemma 1 (**Skimming**). *Consider buyer types with a common history h^t. If a buyer type with value profile v obtains a higher net value along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$, with total consumption $\chi(\mathbf{x}_k^t) \neq \chi(\mathbf{x}_l^t)$, then so does another buyer type with value profile $\tilde{v} \neq v$ such that*

$$(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0. \tag{1}$$

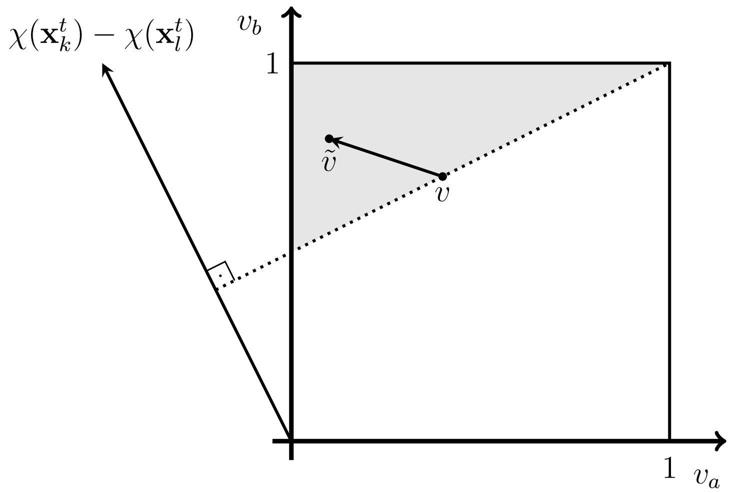
The result states the skimming condition in terms of the (discounted to period t) total consumption levels obtained along two different consumption paths.[12] This condition shows that for two buyer types to have the same preferences over the net values generated along two different consumption paths, the value profiles and the total consumption levels along the two paths must be aligned. That is, it is generally not sufficient for a type to have strictly higher values for both varieties to satisfy the condition; instead, the relative values $(v_a - v_b)$ must be considered. For example, if following path $\mathbf{x}_k^t$ instead of $\mathbf{x}_l^t$ implies obtaining relatively less consumption of a and relatively more consumption of b, and type v chooses path $\mathbf{x}_k^t$ over $\mathbf{x}_l^t$, then only types $\tilde{v}$ who do not prefer a relatively more than b compared to type v will make the same choice. However, if path $\mathbf{x}_k^t$ implies obtaining more consumption of a compared to path $\mathbf{x}_l^t$, while the consumption of b is equal along

[12] An analogous result can be stated in terms of the consumption choices at time t for any history h^t (rather than the total consumption levels obtained along two consumption paths).

the two paths, then $\tilde{v}_a > v_a$ is sufficient for type $\tilde{v}$ to have the same preference. Hence, restricting the set of admissible consumption paths makes it easier to satisfy the skimming condition.

Figure 4 illustrates the skimming condition (1), which characterizes a dot product and restricts the angle between the two direction vectors $(\tilde{v} - v)$ and $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ to a maximum of 90°. For the particular difference $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ depicted, only value profiles in the shaded area (including $(v, \tilde{v})$) satisfy the condition.

Figure 4: Illustration of the skimming condition (1)



3.2 Profit

Any strategy profile $\{\sigma, \hat{\sigma}\}$ gives rise to sequences of prices and consumption choices that can be computed recursively. Let $\mathcal{P}(\mathbf{x}^t | h^t, \sigma, \hat{\sigma}, v)$ be the probability measure that buyer type v chooses consumption path $\mathbf{x}^t \in \mathbf{X}(h^t)$ after history h^t given the strategy profile $\{\sigma, \hat{\sigma}\}$. We can then measure buyer types with the same history h^t, following the same consumption path $\mathbf{x}^t$, as

$$\mathcal{S}\left(\{\mathbf{x}^t | h^t, \sigma, \hat{\sigma}\}\right) = \int_{v \in V(h^t)} \mathcal{P}\left(\{\mathbf{x}^t | h^t, \sigma, \hat{\sigma}, v\}\right) d\mathcal{F}, \text{for } \mathbf{x}^t \in \mathbf{X}(h^t).$$

Accordingly, the seller cannot distinguish between these types.

The seller's present discounted (future) profit at history h^t can be expressed in terms of the strategy-contingent payments made along the admissible consumption paths $\mathbf{x}^t \in \mathbf{X}(h^t)$ by the appropriate measures of buyer types,

$$\Pi(\sigma, \hat{\sigma}|h^t) = \int_{\mathbf{x}^t \in \mathbf{X}(h^t)} \rho(\mathbf{p}^t, \mathbf{x}^t|\sigma, \hat{\sigma}, h^t) d\mathcal{S}\left(\mathbf{x}^t|\sigma, \hat{\sigma}, h^t\right), \tag{2}$$

where $\rho(\mathbf{p}^t, \mathbf{x}^t|\sigma, \hat{\sigma}, h^t)$ is the total strategy-contingent payment along consumption path $\mathbf{x}^t$ after history h^t, and $\mathcal{S}(\mathbf{x}^t|\sigma, \hat{\sigma}, h^t)$ is the strategy-contingent measure of types that follow consumption path $\mathbf{x}^t$ after history h^t.

3.3 Pricing without trading-up opportunities

Borrowing terminology from Board and Pycia [2014], we say that the seller and the buyer follow *monopoly strategies* $\{\sigma^m, \hat{\sigma}^m\}$ if, in every period t, the seller plays static monopoly prices p^m and the buyer behaves as if she were myopic. That is, in every period t,

(i) the seller charges p_i^m for rental variety $i \in \{a, b\}$, and the buyer purchases rental variety i if $v_i - p_i^m \geq \max\{v_j - p_j^m, 0\}$, $j \neq i$;

(ii) the seller charges $p_i^m \Delta^t$ for durable variety $i \in \{a, b\}$ if the buyer has not yet purchased variety i (and zero otherwise), and the buyer purchases durable variety i if $(v_i - p_i^m)\Delta^t \geq \max\{(v_j - p_j^m)\Delta^t, 0\}$, $j \neq i$.

We can then state the following result.

Proposition 1 (Repeated static monopoly). *Suppose there are no trading-up opportunities in the static monopoly outcome, that is,* $p^m \in \Omega$*. Then, in any PBE,*

(i) the seller can do no better than follow the monopoly strategy along the equilibrium path.

(ii) the seller's profit equals the (present discounted) sum of the repeated static monopoly profit, that is, $\Pi = \pi(p^m)\Delta$*.*

Proposition 1 shows that the seller can do no better than follow a monopoly strategy if there are no trading-up opportunities in the static monopoly outcome, regardless of whether durable, rental, or mixed varieties are offered. Intuitively, the result follows because the absence of trading-up opportunities in the static monopoly outcome implies that the seller cannot benefit from inducing the buyer to switch to an alternative consumption path: the resulting short-term loss relative to the static monopoly profit cannot be recouped in the future, because future prices cannot lead to a higher profit than the static monopoly prices that leave no trading-up opportunities.

Importantly, the result implies that Coasian dynamics do *not* emerge in settings without trading-up opportunities in the static monopoly outcome, irrespective of the seller's commitment ability. Hence, it suffices to work out the static monopoly outcome to determine the outcome of the dynamic game for these settings. To be sure, Tirole [2016] has already established this result for a positive-selection setting with a single rental variety and an absorbing outside option with value zero, where trading-up opportunities in static monopoly are excluded by construction.[13] Proposition 1 shows that the result generalizes to settings with multiple varieties under appropriate assumptions. For instance, if the initial state is non-absorbing and the outside option is absorbing, then the result emerges only if the initial state (i.e., a rental variety) is also the most-preferred state of all buyer types in the support (otherwise, there would be trading-up opportunities).[14] That is, in settings with two rentals or mixed varieties, the rental variety in the initial state must be the most-preferred variety for all buyer types in the support.

Board and Pycia [2014] study a related setting with a single durable variety and a non-absorbing outside option with value zero as the initial state. Clearly, Proposition 1 should not be expected to hold in this setting, because there are trading-up opportunities in the static monopoly outcome. Nevertheless, these authors show that Coasian dynamics do not emerge if all buyer types have costless access to *another* absorbing outside option with strictly positive value. Although

[13]See Figure 3b for an illustration of positive selection with a single variety a.

[14]The result trivially emerges if the initial state is absorbing, regardless of the varieties offered by the seller.

outside the scope of our setup, their result is consistent with the intuition for Proposition 1: the additional outside option effectively eliminates trading-up opportunities in the static monopoly outcome. The reason is that all types that do not buy at static monopoly prices prefer the costless additional outside option with strictly positive value to the initial state with value zero, leaving no types in the initial state; hence no trading-up opportunities remain.

3.4 Pricing with trading-up opportunities

We now consider settings with trading-up opportunities in the static monopoly outcome. The classic example is the case of a single durable good with non-absorbing initial state $x^0 = o$, but similar trading-up opportunities emerge with multiple durable, rental, or mixed varieties.

In the case of a single durable good, it is well-known that the seller obtains a strictly positive profit if the lowest valuation is above marginal cost—the so-called "gap case" [Fudenberg and Tirole, 1993, pp. 408]. We first show that this result carries over to settings with multiple durable, rental, or mixed varieties: the seller can obtain a strictly positive profit for certain measures $\mathcal{F}$ of the buyer's value profile by trading up all types at once.

Lemma 2. *If the minimal value of at least one variety is strictly positive, then* $\pi(\bar{p}) > 0$*.*

That is, regardless of whether durable, rental, or mixed varieties are offered, the seller obtains a strictly positive profit if there is a gap for at least one of the varieties.

Next, we show that the seller engages in dynamic pricing after any history at which trading-up opportunities exist until all trading-up opportunities are exhausted. We characterize these pricing dynamics and provide conditions under which they are played out in finite time.

Proposition 2 (Pricing dynamics). *Suppose there are trading-up opportunities in the static monopoly outcome. Then, in any PBE,* *(i) the seller trades up a positive measure of types along the equilibrium path after any history h^t with trading-up opportunities.*

(ii) the seller will never set prices below $\bar{p}$ after any history h^t, implying that the seller's (present discounted) profit satisfies $\Pi \geq \pi(\bar{p})\Delta$.

(iii) all trading-up opportunities are exhausted before the game ends if the minimal value of at least one variety is strictly positive, and T is sufficiently large.

Proposition 2 demonstrates that the driving force behind pricing dynamics is the existence of trading-up opportunities. For a seller who faces trading-up opportunities and lacks commitment, it is profitable to trade up positive masses of types to higher-valued consumption options, thereby extracting a larger surplus. Consequently, the seller changes prices to trade up more and more types as the game progresses. Since no price profile $p \in \Omega$ leaves any trading-up opportunities, neither in the static nor in the dynamic game, the seller does not sell at prices below $\bar{p}$. Hence, the dynamics come to an end at prices $\bar{p}$, provided that transitions to the respective consumption options are admissible.[15] This implies that the seller's profit in the absence of commitment is bounded from below at $\pi(\bar{p})\Delta$, which may be strictly positive. The time it takes for the price dynamics to play out depends on the setting under study. However, for all trading-up opportunities to be exhausted before the game ends, the minimum value for at least one variety must be strictly positive, and the number of periods of play must be sufficiently large.

To understand the intuition for statement (iii), observe that since it is optimal for the seller to engage in trading-up at any history with trading-up opportunities, the seller must decide whether to trade up some or *all* types. The more types the seller has already traded up in previous periods, the smaller is the extra surplus that can be extracted from the remaining types who can still be traded up. Eventually, it no longer pays for the seller to delay the trading up of some lower-value types in order to trade up higher-valued types earlier on at higher prices, and the seller trades up all remaining buyer typs instantaneously. But for this to occur before the game ends, the seller must be able to strictly increase profit by trading

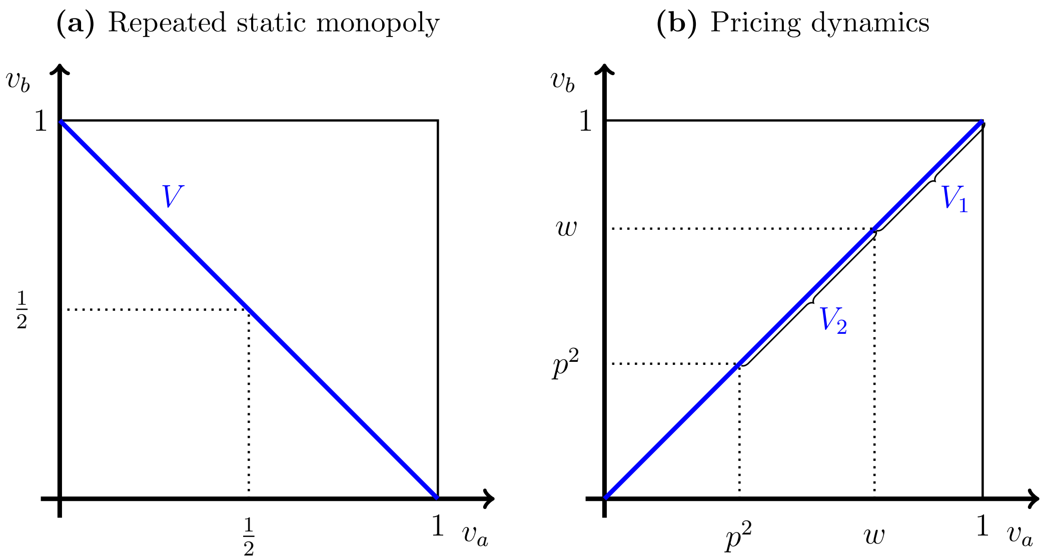
[15]Note that any price is optimal for a non-admissible consumption option.

up all types at once. That is, if the minimum value of at least one variety is strictly positive and there are sufficiently many periods of play, the seller will trade up all types in finite time. Otherwise—in the "no gaps case"—the pricing dynamics may continue indefinitely.

4 Examples

We present two examples with linear support to illustrate Propositions 1 and 2. Specifically, we consider $T = 2$ periods and let the initial state be the non-absorbing outside option, $x^0 = o$. For each example, we examine three different settings: (i) Two durables, (ii) two rentals; and (iii) mixed varieties.[16]

Figure 5: Two examples with linear support



For the first example, assume that buyer types are uniformly distributed on $V = \{v \in [0,1]^2 | v_a + v_b = 1\}$, that is, $\mathcal{F}(E) = \int_E (1/\sqrt{2}) d\mu(v)$, for $E \in \mathcal{B}(V)$, where μ is the Lebesgue measure. We derive the following result in Appendix B, which is illustrated in the left panel of Figure 5.

[16]An example with full support and a bivariate uniform distribution on $V = [0,1]^2$ is available from the authors upon request.

Example 1 (**Repeated static monopoly**; $T = 2$)**.** *Let $x^0 = o$ and assume that buyer types are uniformly distributed on $V = \{v \in [0,1]^2 | v_a + v_b = 1\}$. Then, for two durables, two rentals, or mixed varieties, there exists a PBE where the repeated static monopoly outcome emerges and all buyer types are traded-up in $t = 1$.*

Regardless of the setting, by setting prices such that $p_a^1 + p_b^1 \geq 1$, the supplier can split the set of buyer types into two separate segments that give rise to independent demands (i.e., the demand for variety i is not affected by the price of variety $j \neq i$). In both segments, the buyer's valuation is uniformly distributed on $[\frac{1}{2}, 1]$, and the static monopoly price profile is $p^m = (\frac{1}{2}, \frac{1}{2})$. Example 1 is reminiscent of the introductory example in Tirole [2016] in that there are no trading-up opportunities in the static monopoly outcome, such that no pricing dynamics emerge in the repeated game.

Note that, for a durable variety, the supplier sets the price in the first period to $(1+\delta)/2$; for a rental variety, the supplier sets the price equal to $1/2$ in each period. That is, while the same net present value of the payment emerges, the buyer pays upfront for a durable variety and repeatedly for a rental variety. All buyer types buy in the first period, that is, $\mathcal{F}(V(\{o, p^1, a\})) = \mathcal{F}(V(\{o, p^1, b\})) = 1/2$, and $\mathcal{F}(V(\{o, p^1, o\})) = 0$.

For the second example, assume that buyer types are uniformly distributed on $V = \{v \in [0,1]^2 | v_a = v_b\}$, that is, $\mathcal{F}(E) = \int_E (1/\sqrt{2}) d\mu(v)$, for $E \in \mathcal{B}(V)$, where μ is again the Lebesgue measure. Thus, a and b are perfect substitutes for all buyer types. We derive the following result in Appendix B, which is illustrated in the right panel of Figure 5.

Example 2 (**Pricing dynamics**; $T = 2$)**.** *Let $x^0 = o$ and assume that buyer types are uniformly distributed on $V = \{v \in [0,1]^2 | v_a = v_b\}$. Then, for two durables, two rentals, or mixed varieties, there exists a PBE where types $v \in V_1 = \{v | v_a = v_b \geq w\}$ are traded up in $t = 1$, and types $v \in V_2 = \{v | w \geq v_a = v_b \geq p^2\}$ are traded up in $t = 2$, with cut-off value $w = (2+\delta)/(4+\delta)$ and $p^2 = w/2$.*

Regardless of the setting, the prices in period 2 are the same for first-time buyers/renters, and the same allocation emerges: The buyer buys the higher-valued variety within two periods or abstains from buying altogether. However, the prices in period 1 differ across settings: the price of a durable variety equals the present discounted value of the prices paid for a rental variety over two periods. Thus, the price of the durable variety includes future values; the price of the rental variety, by contrast, allows for a per-period treatment.

Formally, the price of the rental variety in period 1 is $p_i^1 = w(2-\delta)/2$, which is below the threshold type w: some buyer types strategically delay their consumption. Thus, there are trading-up opportunities for the seller and pricing dynamics emerge.

Note that profit-maximizing prices of the static game are $p^m = (1/2, 1/2)$. Thus, half of the buyer types refrain from buying, resulting in a profit of 1/4, and leaving trading-up opportunities for types $v_i \leq 1/2$.

5 Transitional games

In this section, we show how our analysis can be extended to settings in which one of the states is only indirectly accessible from the initial state via another state (e.g., if the buyer can purchase a product only after having leased it). We dub this class of settings "transitional games." Figure 6 provides an example of a transitional game with full support where absorbing variety b is only indirectly accessible via rental variety a from the initial state $x^0 = o$. States that are only indirectly accessible from the initial state pose a challenge for our analysis, because the consumption options accessible in the static game do not correspond to those in the dynamic game, such that there might be no prices that exhaust all trading-up opportunities in the static game.

We now show that our approach of characterizing the dynamic equilibrium by analyzing the associated static game can nevertheless be applied to transitional games by introducing a suitably constructed "extended static game."

Definition 2 (**Extended static game**)**.** *Consider a transitional game* $(x^0 = o, \Gamma, \mathcal{F})$*, where* b *is only indirectly accessible via* a*, from the initial state* $x^0 = o$*. In the associated extended static game, let the probability measure of the buyer's valuation profile be the pushforward measure* $\mathcal{F}^e(E) := \mathcal{F}(l^{-1}(E))$*, for all* $E \in$

$\mathcal{B}(V^e)$, *following from the linear transformation of the support* $l : V \to V^e$, *where*

$$l(v) = \begin{pmatrix} 1 & 0 \\ \frac{1}{\Delta} & \frac{\Delta-1}{\Delta} \end{pmatrix} v, \;\; v \in V.$$

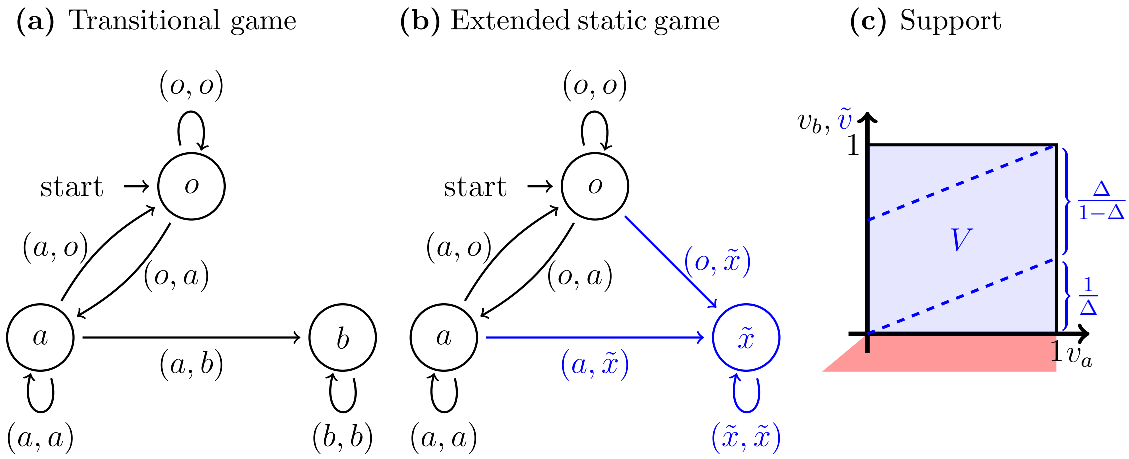
Moreover, let the extended set of admissible transitions be Γ^e.

Thus, in the extended static game, the buyer chooses between the directly accessible states $\{x^0, a\}$ and a virtual state $\tilde{x}$ with value $\tilde{v} = v_a/\Delta + v_b(\Delta - 1)/\Delta$. The value of the virtual state corresponds to the average per period value if the buyer first chooses a and b afterwards, i.e., the shortest way to reach b.

Equivalent to our analysis above, let the static profit of the extended game be $\pi^e(p) = p_a \mathcal{F}^e(v \in V^e | v_a - p_a \geq \max\{v_b - p_b, 0\}) + p_b \mathcal{F}^e(v \in V^e | v_b - p_b \geq \max\{v_a - p_a, 0\})$ and denote the monopoly price by $p^{m,e} \in \arg\max \pi^e(p)$. Moreover, let the set of price profiles that induce an allocation which leaves no trading-up opportunities for the seller in the extended static game be

$$\Omega^e = \left\{ p \in [\psi, 1]^2 \middle| \mathcal{F}^e \left(\left\{ v \in V^e(\{x^0, p^1, x^1\}) \;\middle|\; x^1 \notin \arg\max_{x \in X(h^2)} v \cdot x \right\} \right) = 0 \right\}.$$

Figure 6: A transitional game with associated extended static game



The construction of the extended static game is illustrated in Figure 6. Panel (a) shows the transitional game in its original form, where state b is only indirectly accessible from the initial state o via state a. Panel (b) shows the extended static game where state b is replaced by the virtual state $\tilde{x}$ (indicated in blue), which is accessible via the direct transition $(o, \tilde{x})$. Note that the value of the virtual option $\tilde{v}$ is different from v_b for all off-diagonal value profiles in the support. The support of the value profiles $(v_a, \tilde{v}) \in V^e$ in the extended static game follows from the linear transformation l of the original support V and lies inside the dashed lines in panel (c).

We denote the price profile associated with the profit maximum that leaves no trading-up opportunities in the extended static game by $\bar{p}^e$. Crucially, setting prices to zero in the extended static game leaves no trading-up opportunities, and hence $\bar{p}^e$ always exists. We can then define a set of prices $\tilde{p} = (p_a^1, p_a^2, p_b^2)$ that satisfy the following two restrictions

$$p_a^1 + (\Delta - 1)p_a^2 = \bar{p}_a^e \Delta, \tag{3}$$

$$p_a^1 + (\Delta - 1)p_b^2 = \bar{p}_{\tilde{x}}^e \Delta. \tag{4}$$

The monopoly strategy for the seller, σ^m, then consists of playing prices $\tilde{p}$ in their respective states, i.e. p_a^1 in state x^0 and p_a^2, p_b^2 in states a, b at any period and history, analogous to the seller playing the same static optimal prices repeatedly in the monopoly strategy defined in subsection 3.3. We can then state the following result that shows how our approach of examining trading-up opportunities in static monopoly carries over to transitional games.

Proposition 3 (**Transitional game**). *Consider a transitional game* $(x^0 = o, \Gamma, \mathcal{F})$*, and suppose there are no trading-up opportunities in the associated extended static monopoly outcome, that is,* $p^{m,e} \in \Omega^e$*. Then, in any PBE,*

(i) the seller can do no better than follow the monopoly strategy along the equilibrium path.

(ii) the seller's profit equals the (present discounted) sum of the repeated static monopoly profit of the extended static game, that is, $\Pi = \pi^e(p^{m,e})\Delta$.

Proposition 3 implies that the key insights of our main analysis also apply to transitional games, including our approach of checking for trading-up opportunities in the associated (extended) static game in order to characterize the outcome of the repeated game. However, a subtle difference arises with transitional games: rather than excluding pricing dynamics, $\pi^e(p^{m,e}) = \pi^e(\bar{p}^e)$ allows for a one-time price change in the dynamic game, which accounts for the change in available consumption options after the first period. Hence, the prices associated with the monopoly strategy of the seller, $\tilde{p}$, must differentiate between the first period and subsequent periods and allow for a one-time price change of consumption option a.

Figure 6 illustrates a setting with a one-time price increase. Just as in the setting with mixed varieties, the seller can set a price of zero for the virtual option $\tilde{x}$ and a strictly positive price for the rental variety a, thereby ensuring that no trading-up opportunities are left while achieving a positive profit for the given support, $\pi^e(\bar{p}^e) > 0$. But to implement this in the dynamic game, the seller must set a price of zero for variety a in the first period, since playing a negative price for variety b in future periods cannot constitute a PBE. In effect, the seller is accepting zero profit in the first period to move back from a transitional game to a setting where all buyer types can directly access both states.

6 Conclusion

We have studied a unified analytical framework that captures a broad class of dynamic monopoly pricing problems, including multiple durable, multiple rental, or a mix of varieties. Our analysis demonstrates that the driving force behind pricing dynamics is the existence of trading-up opportunities.

In particular, we show that the emergence of price dynamics hinges on whether the monopoly outcome in the corresponding static game leaves trading-up opportunities to the seller. If there are no trading-up opportunities, then the seller can implement the repeated static monopoly outcome regardless of commitment. Instead, with trading-up opportunities, the seller lowers prices to trade up positive masses of buyer types to higher-valued consumption options until all trading-up opportunities are exhausted or the game ends. Our analysis shows that the essence of Coase's insight generalizes beyond the durable goods case: pricing dynamics emerge whenever some buyer types have access to a higher-valued consumption option.

Our findings imply that dynamic monopoly pricing problems can be analyzed by checking for trading-up opportunities in the static monopoly outcome. This approach also works for transitional games, where one of the varieties is only indirectly accessible from the initial state, provided that the associated extended static games are properly defined.

There is ample scope for future research. In particular, it would be interesting to study the seller's endogenous choice among alternative modes of trade.

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A Proofs

A.1 Proof of Lemma 1

Since type v obtains a higher total value along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$ by assumption, we must have

$$\nu(v, \mathbf{x}_k^t) - \rho(\mathbf{p}_k^t, \mathbf{x}_k^t) \geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{p}_l^t, \mathbf{x}_l^t).$$

Now, consider some type $\tilde{v} \neq v$. Then, we have

$$\begin{aligned}\nu(\tilde{v}, \mathbf{x}_k^t) - \rho(\mathbf{p}_k^t, \mathbf{x}_k^t) &= \nu(v, \mathbf{x}_k^t) - \rho(\mathbf{p}_l^t, \mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \\ &\geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{p}_l^t, \mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t),\end{aligned}$$

since type v obtains a higher total value along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$ by assumption. For type $\tilde{v}$ to obtain a higher total value along path $\mathbf{x}_k^t$, it is thus sufficient to have

$$\nu(v, \mathbf{x}_l^t) - \rho(\mathbf{p}_l^t, \mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \geq \nu(\tilde{v}, \mathbf{x}_l^t) - \rho(\mathbf{p}_l^t, \mathbf{x}_l^t),$$

which can be rearranged to yield the result in (1).

A.2 Proof of Proposition 1

We prove the result by showing that (i) the maximum profit for the seller is the repeated static monopoly profit, and (ii) when facing prices p^m, buyers behave as if they were myopic, implying that the seller can do no better than following the monopoly strategy.

(i) Recall from (2) that the seller's present discounted profit at history h^t is

$$\Pi(h^t) = \int_{\mathbf{x}^t \in \mathbf{X}(h^t)} \rho(\mathbf{p}^t, \mathbf{x}^t | h^t) d\mathcal{S}\left(\mathbf{x}^t | h^t\right), \tag{5}$$

where the strategy profile $\{\sigma, \hat{\sigma}\}$ is suppressed from the arguments for ease of notation. The following auxiliary result shows that $\Pi(h^t)$ can also be expressed in terms of the differences in the total values obtained by buyer types that are indifferent between different consumption paths. Letting

$$\Delta\nu(\mathbf{x}_k^t, \mathbf{x}_{k-1}^t|v_k) \equiv \nu(v_k, \mathbf{x}_k^t) - \nu(v_k, \mathbf{x}_{k-1}^t) = \rho(\mathbf{p}_k^t, \mathbf{x}_k^t) - \rho(\mathbf{p}_{k-1}^t, \mathbf{x}_{k-1}^t), \quad (6)$$

denote the difference in the total values obtained by buyer types v_k that are indifferent between consumption paths $\mathbf{x}_k^t$ with price path $\mathbf{p}_k^t$ and $\mathbf{x}_{k-1}^t$ with $\mathbf{p}_{k-1}^t$, $k \geq 1$, if v_k exists,[17] we can restate the seller's present discounted profit at history h^t as follows:

Lemma 3. *The seller's present discounted profit at history h^t with measure $\mathcal{F}(V(h^t))$ of active buyer types can be expressed as*

$$\Pi(h^t) = \rho(\mathbf{p}_0^t, \mathbf{x}_0^t|h^t)\mathcal{F}(V(h^t)) + \int_{k\geq 1} \Delta\nu(\mathbf{x}_k^t, \mathbf{x}_{k-1}^t|v_k) d\mathcal{S}(\{\mathbf{x}_j^t \in \mathbf{X}(h^t)|j \geq k\}),$$

where $\mathcal{S}(\mathbf{x}_j^t|h^t)$ is the measure of active buyer types that follow consumption path $\mathbf{x}_j^t$ after history h^t, and admissible consumption paths $\mathbf{X}(h^t)$ are well-ordered with respect to $\preceq$ such that $(\mathbf{X}(h^t), \preceq) = \{\mathbf{x}_0^t, \mathbf{x}_1^t, ...|h^t\}$.

Proof. First, because $\mathbf{X}(h^t)$ is finite, there exists a well-order $(\mathbf{X}(h^t), \preceq) = \{\mathbf{x}_0^t, \mathbf{x}_1^t, ...|h^t\}$. Now, consider the first two paths $\mathbf{x}_1^t \neq \mathbf{x}_0^t$. Any indifferent type v_1 that obtains the same total payoff from both paths satisfies the indifference condition

$$\nu(v_1, \mathbf{x}_1^t) - \rho(\mathbf{p}_1^t, \mathbf{x}_1^t) = \nu(v_1, \mathbf{x}_0^t) - \rho(\mathbf{p}_0^t, \mathbf{x}_0^t)$$

or, equivalently,

$$\rho(\mathbf{p}_1^t, \mathbf{x}_1^t) = \rho(\mathbf{p}_0^t, \mathbf{x}_0^t) + \Delta\nu(\mathbf{x}_1^t, \mathbf{x}_0^t|v_1).$$

Similarly, for paths $\mathbf{x}_2^t \neq \mathbf{x}_1^t$, we have

$$\rho(\mathbf{p}_2^t, \mathbf{x}_2^t) = \rho(\mathbf{p}_1^t, \mathbf{x}_1^t) + \Delta\nu(\mathbf{x}_2^t, \mathbf{x}_1^t|v_2)$$

for any indifferent type v_2. By substituting from above we obtain

$$\rho(\mathbf{p}_2^t, \mathbf{x}_2^t) = \rho(\mathbf{p}_0^t, \mathbf{x}_0^t) + \Delta\nu(\mathbf{x}_2^t, \mathbf{x}_1^t|v_2) + \Delta\nu(\mathbf{x}_1^t, \mathbf{x}_0^t|v_1).$$

[17]Note that with profit-maximizing prices, at least one indifferent type v_k exists.

Iterating this procedure, for an arbitrary path $\mathbf{x}_k^t \in (\mathbf{X}(h^t), \preceq)$ we obtain

$$\rho(\mathbf{p}_k^t, \mathbf{x}_k^t) = \rho(\mathbf{p}_0^t, \mathbf{x}_0^t) + \sum_{l=1}^{k} \Delta\nu(\mathbf{x}_l^t, \mathbf{x}_{l-1}^t | v_l).$$

Plugging in to (5), we obtain

$$\Pi(h^t) = \rho(\mathbf{p}_0^t, \mathbf{x}_0^t | h^t)\mathcal{F}\left(V(h^t)\right) + \int_{k\geq 1} \sum_{l=1}^{k} \Delta\nu(\mathbf{x}_l^t, \mathbf{x}_{l-1}^t | v_l) d\mathcal{S}(\{\mathbf{x}_k^t \in \mathbf{X}(h^t)\}),$$

which can be rewritten as

$$\Pi(h^t) = \rho(\mathbf{p}_0^t, \mathbf{x}_0^t | h^t)\mathcal{F}\left(V(h^t)\right) + \int_{k\geq 1} \Delta\nu(\mathbf{x}_k^t, \mathbf{x}_{k-1}^t | v_k) d\mathcal{S}(\{\mathbf{x}_j^t \in \mathbf{X}(h^t) | j \geq k\}).$$

□

Now, suppose that both the seller and the buyer follow a monopoly strategy. Then, using Lemma 3, the seller's profit from $t = 1$ onward is given by

$$\Pi(h^1) = \rho(\mathbf{p}^m, \mathbf{x}_a | h^1) + \Delta\nu(\mathbf{x}_b, \mathbf{x}_a | v_1)\mathcal{S}(\mathbf{x}_b | h^1), \tag{7}$$

where $\mathbf{p}^m = \{p^m, p^m, p^m, ...\}$, $\mathbf{x}_a = \{a, a, a, ...\}$ and $\mathbf{x}_b = \{b, b, b, ...\}$, respectively. Because there are no trading-up opportunities, we must have $\mathcal{S}(\mathbf{x}_k | h^1) = 0$ for any path $\mathbf{x}_k \neq \mathbf{x}_i$ for which $\Delta\nu(\mathbf{x}_k, \mathbf{x}_i | v_k) > 0$, $i = a, b$. Hence, the repeated static monopoly profit in (7) is the maximum profit.

(ii) First, observe that at any history h^t all types choose their most-preferred variety when facing price profile $p^\circ = (-\Delta^t, -\Delta^t)$. To see this, note that types v and $\tilde{v}$, $v \neq \tilde{v}$, can always mimic each other's behavior (i.e., make the same choices from t onward), so that we have

$$U(\tilde{v}, x^t, h^t) - U(v, x^t, h^t) \leq \max_{i\in\{a,b\}} \{\tilde{v}_i - v_i\}\Delta^t, \quad v \neq \tilde{v},$$

where U denotes the continuation valuation following choice x^t. Since the maximum value difference satisfies $\max_{i\in\{a,b\}}\{\tilde{v}_i - v_i\} = 1$, all types purchase their most-preferred variety when facing prices p°. In addition, it is straightforward that when facing p° in the static game, all types will equally accept and purchase their most-preferred variety so that $p^\circ \in \Omega$.

Now pick a price profile $\tilde{p}$ on the diagonal through the type space that satisfies

$$\tilde{p} = (\min\{p_a^m, p_b^m\}, \min\{p_a^m, p_b^m\}) - (\eta, \eta),$$

for some $0 \leq \eta \leq \Delta^t + \min\{p_a^m, p_b^m\}$. It is straightforward that all types $\max_{i \in \{a,b\}}\{v_i\} \geq \tilde{p}_i$ will accept and purchase their most-preferred variety when facing $\tilde{p}$ in the static game, implying that $\tilde{p} \in \Omega$ since $p^m \in \Omega$.

Denote by x° the choice that buyers make in the static game when facing prices p°. We therefore have

$$x^\circ \cdot (v - p^\circ) \geq x' \cdot (v - p^\circ), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \tag{8}$$

where we know that $x^\circ \in \{a, b\}$ as $p_a^\circ = p_b^\circ < 0$. And, similarly, we also have that

$$\begin{aligned} &x^\circ \cdot (v - p^\circ) + \delta U(v, x^\circ, h^t) \\ &\qquad \geq x' \cdot (v - p^\circ) + \delta U(v, x', h^t), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V. \end{aligned} \tag{9}$$

By the definition of p° it then follows that

$$\delta(U' - U^\circ) \leq (x^\circ - x') \cdot v, \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \tag{10}$$

where U° and U' denote the continuation valuations associated with the choices x° and x' respectively, given history h^t. This directly implies that

$$\begin{aligned} &x^\circ \cdot (v - \tilde{p}) + \delta U(v, x^\circ, h^t) \\ &\qquad \geq x' \cdot (v - \tilde{p}) + \delta U(v, x', h^t), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \end{aligned}$$

that is, all types behave as if they were myopic when facing $\tilde{p}$ at any history h^t. This suffices to prove the statements if $p_a^m = p_b^m$.

To complete the proof for the case of $p_a^m \neq p_b^m$, consider a price profile

$$\hat{p} = \begin{cases} (\min\{p_a^m, p_b^m\}, \min\{p_a^m, p_b^m\}) + (0, \varepsilon) & \text{if } p_b^m > p_a^m \\ (\min\{p_a^m, p_b^m\}, \min\{p_a^m, p_b^m\}) + (\varepsilon, 0) & \text{if } p_b^m < p_a^m \end{cases} \tag{11}$$

where $\varepsilon \in [0, \max\{p_a^m, p_b^m\} - \min\{p_a^m, p_b^m\}]$. Then by the same logic as above, we find that when facing $\hat{p}$ compared to $\tilde{p}$, the only types that now prefer the outside option to consumption also prefer the outside option at prices p^m and the only types now preferring their non-most-preferred variety also do so at prices p^m. Thus, we find that all types follow the monopoly strategy at any history h^t when facing prices p^m.

Then it follows that the seller can do no better than following the monopoly strategy along the equilibrium path, achieving the present discounted sum of the repeated static monopoly profit, as this is the highest possible profit as shown above.

A.3 Proof of Lemma 2

Let j be the variety with a strictly positive minimal value, i.e., $\underline{v}_j > 0$. Suppose the seller sets prices $(p_a, p_b) = (\underline{v}_j, \underline{v}_j)$. Consider the static setup and note that with these prices all types prefer variety j to the outside option o. Moreover, because prices are equal for both varieties, buyer types prefer variety i to j if and only if $v_i \geq v_j$. Therefore, buyer types buy their most-preferred variety if accessible and no trading-up opportunities remain, i.e., $(\underline{v}_j, \underline{v}_j) \in \Omega$.

Next, note that the seller obtains a profit of $\underline{v}_j$ in the static game with prices $(p_a, p_b) = (\underline{v}_j, \underline{v}_j)$, because every buyer type buys an accessible variety. Therefore, by definition $\pi(\bar{p}) \geq \underline{v}_j > 0$.

A.4 Proof of Proposition 2

We prove the three statements in turn.

(i) Fix a PBE and consider a history h^t with trading-up opportunities, such that $\mathcal{F}\left(\{v \in V(h^t)\ |x^{t-1} \notin \arg\max_{x \in X(h^t)} v \cdot x\}\right) > 0$. Let $\bar{v}_i = \max_{v \in V(h^t)} v_i$ be the highest value for variety $i \in \{a, b\}$ of all types with this history, and let $\underline{v}_i = \min_{v \in V(h^t)} v_i$ be the lowest value. A type $v \in V(h^t)$ can be traded up at history h^t if $v \cdot x^t > v \cdot x^{t-1}$ and $x^t \in X(h^t)$. We denote the highest and lowest value of a type with history h^t that can be traded up by $\bar{v}_i^{TU}$ and $\underline{v}_i^{TU}$, respectively.

Further denote the mass of types that are traded up at history h^t by $\mathcal{M}^{TU}(h^t) = \mathcal{F}(\{v \in V(h^t) \mid v \cdot x^t > v \cdot x^{t-1}\})$ and the remaining mass of types that are not traded up by $\mathcal{M}^{NTU}(h^t)$, such that $\mathcal{F}(V(h^t)) = \mathcal{M}^{TU}(h^t) + \mathcal{M}^{NTU}(h^t)$. We will show that, for any candidate equilibrium path starting at history h^t along which the seller does not trade up any types, trading up a positive measure of types instead is strictly profit-increasing. There are three cases to distinguish.

Case 1: $x^{t-1} = o$, and $\{a, b\} \cap X(h^t) \neq \emptyset$.
If x^{t-1} is the outside option, the existence of trading-up opportunities implies that the seller can induce a positive measure of types $\mathcal{M}^{TU}(h^t)$ to buy variety $i \in \{a, b\}$ with $i \in X(h^t)$ at strictly positive prices, which is profit-increasing.

Case 2: $x^{t-1} = j \in \{a, b\}$, $i \in X(h^t), i \neq j$, and $\bar{v}_i^{TU} > \bar{v}_j$.
For trading-up opportunities to exist, x^{t-1} must be a non-absorbing variety j. Then the equilibrium profit from not trading up any types, $\hat{\Pi}(h^t)$, satisfies

$$\hat{\Pi}(h^t) < \bar{v}_j \mathcal{F}(V(h^t))\Delta^t, \tag{12}$$

as the seller cannot extract the full surplus from all types v at history h^t with a linear price. If the seller trades up the mass of types $\mathcal{M}^{TU}(h^t)$, then the equilibrium profit obtained from types traded up, $\Pi^*(h^t)$, satisfies

$$\Pi^*(h^t) \geq v_i^* \mathcal{M}^{TU}(h^t)\Delta^t, \tag{13}$$

where v_i^* denotes the lowest value v_i of the cutoff types who are indifferent to trading up to i, as the seller can always obtain at least the value of the lowest type in the set. The equilibrium profit obtained from types not traded up, $\Pi^\circ(h^t)$, satisfies

$$\Pi^\circ(h^t) < \bar{v}_j \mathcal{M}^{NTU}(h^t)\Delta^t, \tag{14}$$

because as before the seller cannot extract the full surplus using a linear price. As $\bar{v}_i^{TU} > \bar{v}_j$ by assumption, there exists a v_i^* that satisfies $\bar{v}_i^{TU} > v_i^* > \bar{v}_j$. Therefore, (12), (13), (14), and $\mathcal{F}(V(h^t)) = \mathcal{M}^{TU}(h^t) + \mathcal{M}^{NTU}(h^t)$ together imply that

$$\Pi^*(h^t) + \Pi^\circ(h^t) > \hat{\Pi}(h^t).$$

Case 3: $x^{t-1} = j \in \{a, b\}$, $i \in X(h^t), i \neq j$, and $\bar{v}_i^{TU} < \bar{v}_j$.
Starting at $x^{t-1} = j$, there are three paths of play without trading up any types

to i. We consider these in turn and show that in each case, there are deviation incentives for the seller, that is, trading up (some) types to i is strictly profit increasing.

(a) Suppose the buyer always plays j along the equilibrium path. As all types only ever accept at a price at which they obtain a (weakly) positive utility along the path of play, we must have that the equilibrium profit obtained satisfies

$$\Pi(h^t) \leq \underline{v}_j \mathcal{F}(V(h^t))\Delta^t. \tag{15}$$

As $\bar{v}_i^{TU} > \underline{v}_j$ by the assumption that trading-up opportunities exist, there exists a v_i^* that satisfies $v_i^* > \underline{v}_j$, which by (13) implies that trading up (some) types to i is strictly profit increasing.

(b) Suppose the seller induces some types to play $x^t = o$ and the outside option is non-absorbing. Consider the types v for which $v_i > v_j$. Either, (some of) these types play $x^{t+1} = j$, such that case (a) applies at the continuation history for these types at time $t+1$, or they play $x^{t+1} = o$, such that Case 1 applies at $t+1$. Trading up is strictly profit-increasing in either of these cases.

(c) Suppose the seller induces some types to play $x^t = o$ and the outside option is absorbing. As before, if there exist v that satisfy $v_i > v_j$ that play $x^{t+1} = j$, then case (b) applies. If not however, all types v for which $v_i > v_j$ must have played $x^t = o$ and can no longer be traded up, as the outside option is absorbing. Consider that types with history h^t that satisfy $v_i > v_j$, must have faced some $p_j^{t-1} \leq \underline{v}_j^{TU}$ at time $t-1$, because in equilibrium, no type accepts unless they obtain a weakly positive present-discounted utility. But if at time t the profit-maximizing choice for the seller is to play some $p_j^* \geq \bar{v}_j^{TU}$ at history h^t, then at time $t-1$ playing $p_j^{t-1} \leq \underline{v}_j^{TU}$ cannot be profit-maximizing for types with history h^t. Suppose the seller had instead played p^* at time $t-1$. If this is the profit-maximizing price at h^t, then profit from the buyer types with history h^t at time $t-1$ must be increased and the present-discounted total profit at time $t-1$ must also be increased, since all types with history h^t that do not play $x^{t-1} = j$ at this higher price either play $x^{t-1} = o$, leaving overall profit unaffected, or play $x^{t-1} = i$ yielding a strictly positive profit.

Then in conjunction statement (i) follows.

(ii) The proof follows the same lines as part (ii) of the proof of Proposition 1. We apply this proof structure to the set of all price profiles that leave no trading up opportunities in the static game, Ω. Specifically, denote by Λ the set of price profiles p that leave no trading-up opportunities for any history h^t in the dynamic game. We will now show that $\Omega \setminus \Lambda = \emptyset$ and $\bar{p} \in \Lambda$.

First, note that because $\bar{p} \in \Omega$ by assumption, for a price profile $\tilde{p}$ on the diagonal through the type space, with $\eta \geq 0$, we have

$$\tilde{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\}) - (\eta, \eta) \implies \tilde{p} \in \Omega, \tag{16}$$

as all types willing to purchase at prices $\tilde{p}$ choose their most-preferred variety, and all types choosing the outside option will also do so at prices $\bar{p}$. Similarly, for a price profile $\tilde{\tilde{p}}$ on the (vertical or horizontal) line between $\bar{p}$ and the diagonal, with $\eta \in [0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$, we have

$$\tilde{\tilde{p}} = \begin{cases} (\bar{p}_a, \bar{p}_b) - (0, \eta), & \text{if } \bar{p}_b > \bar{p}_a \\ (\bar{p}_a, \bar{p}_b) - (\eta, 0), & \text{if } \bar{p}_b < \bar{p}_a \end{cases} \implies \tilde{\tilde{p}} \in \Omega, \tag{17}$$

as all types purchasing a different variety at prices $\tilde{\tilde{p}}$ than at prices $\bar{p}$ must now choose their most-preferred variety, and all types switching from the outside option to consumption must now choose their most-preferred variety.

Second, recall from part (ii) of the proof of Proposition 1 that the price profile $p^\circ = (-\Delta^t, -\Delta^t)$ satisfies $p^\circ \in \Lambda$ as all types can always mimic each other's behavior. In addition, by (16) we also have that $p^\circ \in \Omega$.

Now pick a price profile $\hat{p}$ that satisfies $\hat{p} = p^\circ + (\varepsilon, \varepsilon)$ for some $\varepsilon \in [0, \Delta^t + \min\{\bar{p}_a, \bar{p}_b\}]$. By (16) we know $\hat{p} \in \Omega$. Denote by x° the choice that buyers make in the static game when facing prices p°. By (16) we then have

$$x^\circ \cdot (v - p^\circ) \geq x' \cdot (v - p^\circ), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \tag{18}$$

where we know that $x^\circ \in \{a, b\}$ as $p^\circ_a = p^\circ_b < 0$. Since $p^\circ \in \Lambda$, we also have that

$$\begin{aligned} &x^\circ \cdot (v - p^\circ) + \delta U(v, x^\circ, h^t) \\ &\qquad \geq x' \cdot (v - p^\circ) + \delta U(v, x', h^t), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \end{aligned} \tag{19}$$

By the definition of p° and (19) it then follows that

$$\delta(U' - U^\circ) \leq (x^\circ - x') \cdot v, \;\; x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \tag{20}$$

where U° and U' denote the continuation valuations associated with the choices x° and x' respectively, given history h^t. This also implies that

$$\begin{aligned} &x^\circ \cdot (v - p^\circ - \varepsilon) + \delta U(v, x^\circ, h^t) \\ &\qquad \geq x' \cdot (v - p^\circ - \varepsilon) + \delta U(v, x', h^t), \qquad x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V. \end{aligned} \tag{21}$$

Thus, for any $\varepsilon \in [0, \Delta^t + \min\{\bar{p}_a, \bar{p}_b\}]$, only types $v < \min\{\bar{p}_a, \bar{p}_b\}$ may choose o over x° at prices $\hat{p}$, which continues to leave no trading-up opportunities since $\bar{p} \in \Omega$ by assumption, and therefore $\hat{p} \in \Lambda$. Hence, for any $\tilde{p}$ that satisfies (16) we have $\tilde{p} \in \Lambda$.

Now fix the price profile $\hat{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\})$. By (17) we have $\hat{p} \in \Omega$, and as shown above we also have $\hat{p} \in \Lambda$. Consider a price profile $p' = \hat{p} + (0, \varepsilon)$ if $\bar{p}_b > \bar{p}_a$ and $p' = \hat{p} + (\varepsilon, 0)$ if $\bar{p}_b < \bar{p}_a$ where $\varepsilon \in [0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$. Then by the same logic as above, for any $\varepsilon \in [0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$, we find that the only types that may choose the outside option over consumption also choose the outside option at prices $\bar{p}$ and the only types who may choose the other variety also do so at prices $\bar{p}$. Thus, we find $p' \in \Lambda$ or equally that any $\tilde{p}$ that satisfies (17) satisfies $\tilde{p} \in \Lambda$ and therefore $\bar{p} \in \Lambda$.

Finally, note that we can construct (16) and (17) for any price profile $p \in \Omega$ and thus we find that $\Omega \setminus \Lambda = \emptyset$. Then it follows from the definition of $\bar{p}$ that the seller will never play prices below $\bar{p}$, implying that the present discounted stream of profits $\pi(\bar{p})\Delta$ is a lower bound on the sellers' profit.

(iii) Fix a PBE and consider a history h^t with trading-up opportunities, that is, $\mathcal{F}\left(\{v \in V(h^t) \;| x^{t-1} \notin \arg\max_{x \in X(h^t)} v \cdot x\}\right) > 0$. Let $\bar{v}_i(h^t) = \max_{v \in V(h^t)} v_i$ be the highest value for variety $i \in \{a, b\}$ of all types with this history h^t and $\underline{v}_i(h^t) = \min_{v \in V(h^t)} v_i$ be the lowest value. A type $v \in V(h^t)$ can be traded up at history h^t if $v \cdot x^t > v \cdot x^{t-1}$ and $x^t \in X(h^t)$. We denote the highest and lowest value of a type with history h^t that can be traded up by $\bar{v}_i^{TU}(h^t)$ and $\underline{v}_i^{TU}(h^t)$, respectively. We define analogously $\bar{v}_j^{TU}(h^t), \underline{v}_j^{TU}(h^t)$ for $j \in \{a, b\}, j \neq i$, if trading-up opportunities exist for j as well. Further denote the mass of types that can be traded up by $\mathcal{M}^{TU}(h^t) = \mathcal{F}(\{v \in V(h^t) \mid x^{t-1} \notin \arg\max_{x \in X(h^t)} v \cdot x\})$.

Let $\bar{v}^{TU}(h^t) = \max\{\bar{v}_i^{TU}(h^t), \bar{v}_j^{TU}(h^t)\}$ and $\underline{v}^{TU}(h^t) = \min\{\underline{v}_i^{TU}(h^t), \underline{v}_j^{TU}(h^t)\}$ denote the highest and lowest value, respectively, of the varieties that buyers at history h^t can be traded up to. Assume without loss of generality that $\underline{v}_i^{TU}(h^t) \leq \underline{v}_j^{TU}(h^t)$ and consider some $\varepsilon(h^t)$ that satisfies

$$\varepsilon(h^t) \geq \bar{v}^{TU}(h^t) - \underline{v}_i^{TU}(h^t).$$

As the seller trades up a positive measure of buyers at any history h^t with trading-up opportunities (see part (i)), by definition of $\bar{v}^{TU}(h^t)$ we have that $\bar{v}^{TU}(h^t) - \underline{v}_i^{TU}(h^t)$ must decrease with the length of a history by Lemma 1, such that a smaller $\varepsilon(h^t)$ will satisfy the above condition.[18] We now show that for $\varepsilon(h^t)$ small enough, the seller strictly prefers to trade up all types at once if the minimal value of at least one variety and $\pi(\bar{p})$ are strictly positive. To ease notation, we henceforth suppress the conditioning of $\mathcal{M}^{TU}$, ε, $\bar{v}^{TU}$ and $\underline{v}_i^{TU}$ on history h^t whenever possible.

As trading up will occur along the equilibrium path for any history with trading-up opportunities (see part (i)), consider t to be the period at which trading-up is profit increasing for the seller for the given history. Let $\Pi^*(h^t)$ denote the equilibrium profit for the seller obtained from trading up only part of the mass $\mathcal{M}^{TU}$. As the seller cannot extract the full surplus with a linear price or trade up the remaining types before time $t+1$, there exists a $\lambda \in (0, 1)$ such that

$$\Pi^*(h^t) < \lambda \mathcal{M}^{TU} \bar{v}^{TU} \Delta^t + \delta(1-\lambda)\mathcal{M}^{TU} \bar{v}^{TU} \Delta^{t+1}.$$

In addition, let $\bar{\Pi}(h^t)$ denote the seller's equilibrium profit obtained from trading up all types. As the seller can always obtain at least the minimal value of a variety in each period, we have that

$$\bar{\Pi}(h^t) \geq (1-\varphi)\mathcal{M}^{TU} \underline{v}_i^{TU} \Delta^t + \varphi \mathcal{M}^{TU} \underline{v}_j^{TU} \Delta^t, \tag{22}$$

[18] Lemma 1 implies $\bar{v}^{TU}(h^t) > \bar{v}^{TU}(h^{t+1})$ because types with a high valuation are traded up earlier than types with a low valuation. Thus, with T sufficiently large, ε becomes sufficiently small over time.

with $\varphi \in [0,1]$.

Using these profits and noting that $\delta\Delta^{t+1} = \Delta^t - 1$, we can write

$$\begin{aligned}
\Pi^*(h^t) - \bar{\Pi}(h^t) &< \lambda \mathcal{M}^{TU} \bar{v}^{TU} \Delta^t + \delta(1-\lambda)\mathcal{M}^{TU} \bar{v}^{TU} \Delta^{t+1} - \\
&\quad (1-\varphi)\mathcal{M}^{TU} \underline{v}_i^{TU} \Delta^t - \varphi \mathcal{M}^{TU} \underline{v}_j^{TU} \Delta^t \\
&= \left[(\Delta^t - 1 + \lambda)\bar{v}^{TU} - (1-\varphi)\underline{v}_i^{TU}\Delta^t - \varphi \underline{v}_j^{TU} \Delta^t \right] \mathcal{M}^{TU} \\
&\leq \left[(\Delta^t - 1 + \lambda)(\varepsilon + \underline{v}_i^{TU}) - (1-\varphi)\underline{v}_i^{TU}\Delta^t - \varphi \underline{v}_j^{TU} \Delta^t \right] \mathcal{M}^{TU} \\
&= \left[(\Delta^t - 1 + \lambda)\varepsilon - (1-\lambda)\underline{v}_i^{TU} + \varphi \Delta^t (\underline{v}_i^{TU} - \underline{v}_j^{TU}) \right] \mathcal{M}^{TU}.
\end{aligned}$$

Therefore, $\bar{\Pi}(h^t) > \Pi^*(h^t)$ whenever

$$\varepsilon(h^t) \leq \frac{(1-\lambda)\underline{v}_i^{TU}(h^t) + \varphi \Delta^t (\underline{v}_j^{TU}(h^t) - \underline{v}_i^{TU}(h^t))}{\Delta^t - 1 + \lambda}.$$

Recall that $\underline{v}_i^{TU}(h^t) \leq \underline{v}_j^{TU}(h^t)$ and note that the right-hand side is strictly positive if $\underline{v}_i^{TU}(h^t) > 0$.

It remains to be shown that the right-hand side is also strictly positive if $\underline{v}_i^{TU}(h^t) = 0$. To see this, note that $\underline{v}_j^{TU}(h^t) > 0$ because the minimal value of at least one variety is strictly positive by assumption. Hence, it suffices to show that $\varphi > 0$. Suppose instead that $\varphi = 0$. From equation (22) we get that $\bar{\Pi}(h^t) = 0$, which is a contradiction: by Lemma 2, we know that $\underline{v}_j(h^t) > 0$ implies $\pi(\bar{p}) > 0$, from which $\bar{\Pi}(h^t) > 0$ follows by (ii).

A.5 Proof of Proposition 3

We first show that, for any strategy profile $\{\sigma, \hat{\sigma}\}$ of the dynamic game, we can define an associated static game that delivers the same payoffs to all players when multiplied with the (present discounted) number of periods Δ, with the extended static game of Definition 2 as a special case. Recall from (2) that the seller's profit at history h^1 is

$$\Pi(\sigma, \hat{\sigma}|h^1) = \int_{\mathbf{x}^1 \in \mathbf{X}(h^1)} \rho(\mathbf{p}^1, \mathbf{x}^1 | \sigma, \hat{\sigma}, h^1) d\mathcal{S}\left(\mathbf{x}^1 | \sigma, \hat{\sigma}, h^1\right),$$

where $\rho(\mathbf{p}^1, \mathbf{x}^1 | \sigma, \hat{\sigma}, h^1)$ is the strategy-contingent total payment along path $\mathbf{x}^1 \in \mathbf{X}^1(h^1)$ and $\mathcal{S}\left(\mathbf{x}^1 | \sigma, \hat{\sigma}, h^1\right)$ denotes the associated measure of types. Now, let $\bar{\rho}(\mathbf{x}^1)$

denote the per-period payment that, if it were received in every period, would give the seller the same (present-discounted) total profit as consumption path $\mathbf{x}^1$,

$$\bar{\rho}(\mathbf{x}^1) = \frac{\rho(\mathbf{p}^1, \mathbf{x}^1 | \sigma, \hat{\sigma}, h^1)}{\Delta}.$$

Similarly, let $\bar{\nu}(v, \mathbf{x}^1)$ be the per-period consumption value that, if it were received in every period, would give a buyer of type v the same (present-discounted) total value as that obtained along path $\mathbf{x}^1$,

$$\bar{\nu}(v, \mathbf{x}^1) = \frac{\nu(v, \mathbf{x}^1)}{\Delta}.$$

Then, the seller's profit can be written as

$$\Pi(\sigma, \hat{\sigma} | h^1) = \Delta \int_{\mathbf{x}^1 \in \mathbf{X}^1(h^1)} \bar{\rho}(\mathbf{x}^1) d\mathcal{S}\left(\mathbf{x}^1 | \sigma, \hat{\sigma}, h^1\right),$$

whereas the net value obtained by a type-v buyer along path $\mathbf{x}^1$ becomes $\Delta(\bar{\nu}(v, \mathbf{x}^1) - \bar{\rho}(\mathbf{x}^1))$. That is, in the dynamic game associated with strategy profile $\{\sigma, \hat{\sigma}\}$, the seller obtains a fixed per-period payment along all admissible paths $\mathbf{x}^1$, multiplied by the (present discounted) number of periods Δ. Each path $\mathbf{x}^1 \notin \{\mathbf{x}_o, \mathbf{x}_a, \mathbf{x}_b\}$ represents a mixed consumption option.

Now consider the transitional game. Denote the monopoly prices in the associated extended static game by $p^{m,e} = (p_a^{m,e}, p_{\tilde{x}}^m)$. By Proposition 1 (i), we obtain that this is the maximum profit if prices $p^{m,e}$ leave no trading-up opportunities, or $\pi^e(p^{m,e})\Delta = \pi^e(\bar{p}^e)\Delta$.

Assume that this is the case, that is, $p^{m,e} \in \Omega^e$ and consider the monopoly strategy for the seller σ^m in the transitional game that consists of playing the tuple of prices $\tilde{p} = (p_a^1, p_a^2, p_b^2)$ repeatedly in the associated states. By (3) and (4), this strategy yields the maximum profit for the seller if the buyer plays the monopoly strategy. Now observe that (3) and (4) also imply that

$$v_a - p_a^2 \geq v_b - p_b^2 \implies v_a - \bar{p}_a^e \geq \tilde{v} - \bar{p}_{\tilde{x}}^e, \tag{23}$$

which can be seen by substituting p_a^2, p_b^2 from the definitions of $\bar{p}_a^e$ and $\bar{p}_{\tilde{x}}^e$ and $\tilde{v}$ into the indifference condition. That is, prices p_a^2, p_b^2 implement the same indifference condition in the one-shot game in any period $t > 1$ in the transitional game at states a, b as prices $\bar{p}^e_a, \bar{p}^e_{\tilde{x}}$ do in the extended static game.

As the outside option is non-absorbing, it must be that no positive measure of types is allocated to the outside option when facing prices $\bar{p}^e$ in the extended static game. Similarly a is non-absorbing by construction and thus no positive measure of types that prefer b must be allocated to a at any time $t > 1$ and history h^t. By (23) therefore, if no positive measure of types is allocated to the outside option in the transitional game at time $t = 1$, then at time $t = 2$ and history h^2 strategy σ^m implements the same allocation in the one-shot game of the transitional game as prices $\bar{p}^e$ do in the extended static game and thus leaves no trading-up opportunities. We can therefore directly apply part (ii) of the proof of Proposition 1 to prove that at any history of the transitional game at any time $t > 1$ and states a, b, the resulting allocation when playing prices (p^2_a, p^2_b) will (i) leave no trading-up opportunities and (ii) achieve the maximum profit if no positive measure was allocated to the outside option at time $t = 1$.

Thus it only remains to check that playing σ^m also ensures that no positive measure of types is allocated to the outside option at $t = 1$, or

$$(v - p^1_a) \cdot a + \delta U(v, a, h^1) \geq (v - 0) \cdot o + \delta U(v, o, h^1) \; \forall v \in V, \tag{24}$$

where U denotes the continuation utility following a and o respectively. Given strategy $\tilde{\sigma}$, it is straightforward that $U(v, a, h^1) \geq 0 \; \forall v \in V$ by construction. We therefore find that playing $p^1_a \leq \underline{v}_a$ ensures that (24) is satisfied, where $\underline{v}_a = \min_{v \in V} v_a$. Thus, the seller can play $\tilde{\sigma}$ and all types will allocate themselves as they do in $\pi^e(\bar{p}^e)$, while prices satisfy the necessary restrictions to ensure the sellers profit is $\Delta\pi^e(\bar{p}^e)$.

B Examples

B.1 Example 1: Repeated static monopoly

Let $V = \{v \in [0,1]^2 | v_b = 1 - v_a\}$, as illustrated in Figure 5 and $\mathcal{F}(E) = \int_E (1/\sqrt{2}) d\mu(v)$, for $E \in \mathcal{B}(V)$, where μ is the Lebesgue measure.

Let us start with the static monopoly prices. Note that any price profile with $p_a + p_b < 1$ yields a lower profit than $p_a + p_b = 1$. For $p_a + p_b \geq 1$ the demand function for variety $i \in \{a, b\}$ is $d_i(p) = 1 - p_i$; thus, we obtain $\pi(p) = \sum_i (1 - p_i) p_i$ and the resulting static monopoly price profile is $p^m = (1/2, 1/2)$.

Next, consider the set of price profiles that leave no trading-up opportunities in the static game. (i) for two durable varieties, $\Omega = \{p \in [\psi, 1]^2 | p_a + p_b \leq 1\}$; (ii) for two rental varieties, $\Omega = \{p \in [\psi, 1]^2 | p_a = p_b \leq 1/2\}$; (iii) and for mixed varieties, where a is the rental and b is the durable, $\Omega = \{p \in [\psi, 1]^2 | p_b \leq \min\{1 - p_a, p_a\}\}$. We obtain in all three cases $p^m \in \Omega$, thus, no trading-up opportunities exist at the static monopoly price profile.

Next, we derive a PBE for $T = 2$. Let us start with (i) two durables. Suppose only buyer types $v_i \leq w_i, i \in \{a, b\}$, remain in the market in period 2.[19] Any price profile $p_a^2 + p_b^2 < 1$ is not profit maximizing.[20] The demand function in period 2 is, therefore, $d_i(p_i^2) = (w_i - p_i^2)$. Maximizing the profit in the second period subject to $p_a^2 + p_b^2 \geq 1$ yields $p_i^2 = 1/2 + (w_i - w_j)/4$, with $i \neq j \in \{a, b\}$. Next, let $w_i = p_i^1 - \delta p_i^2$ be the cut-off type who strategically delays his purchase.[21] The discounted sum of profits is $\sum_{i \in \{a,b\}} (1 - w_i) p_i^1 + \delta \sum_{i \in \{a,b\}} (w_i - p_i^2) p_i^2$. Note that the cut-off types determine all prices, thus, we can rewrite the supplier's profit function in terms of the cut-off types (w_a, w_b), which determine when to trade-up

[19]buyer type $v = (v_a, v_b)$ buys variety i in period 1 iff $v_i + \delta v_i - p_i^1 \geq \max\{\delta(v_i - p_i^2), v_j + \delta v_j - p_j^1, \delta(v_j - p_j^2), 0\}$. In the symmetric equilibrium, which we are constructing, this condition simplifies to $v_i \geq w_i$.

[20]The supplier can increase prices to $p + (\varepsilon, \varepsilon)$, for $\varepsilon \in (0, (1 - p_a - p_b)/2]$, without affecting the demand resulting in a higher profit.

[21]In a symmetric equilibrium, buyer types purchase their higher-valued variety or none. Therefore, the cut-off type is defined as being indifferent between buying the same variety today or tomorrow.

which types:

$$\sum_{i\in\{a,b\}}(1-w_i)\left(w_i+\delta\left(\frac{1}{2}+\frac{w_i-w_j}{4}\right)\right)+\delta\sum_{i\in\{a,b\}}\frac{3w_i+w_j-2}{4}\left(\frac{1}{2}+\frac{w_i-w_j}{4}\right).$$

The resulting profit maximizing cut-off types are $w_i = 1/2$, implying $p_i^2 = 1/2$ and $p_i^1 = (1+\delta)/2$. Thus, all buyer types buy in $t = 1$ their higher-valued variety.

Next, consider (ii) two rental varieties. The above analysis implies that the price for first-time renters in period 2 is $p_i^2(o) = 1/2 + (w_i - w_j)/4$. The profit that the supplier obtains from these types is thus equal to the profit that the supplier in (i) obtains from buyer types in the second period. The cut-off type is $w_i = p_i^1 + \delta p_i^2(i) - \delta p_i^2(o)$,[22] and the net present value of the price for repeat renters is $p_i^1 + \delta p_i^2(i)$. The supplier obtains the profit $\sum_{i\in\{a,b\}}(1-w_i)(p_i^1+\delta p_i^2(i)) + \delta\sum_{i\in\{a,b\}}(w_i - p_i^2)p_i^2$, which is equivalent to the profit in (i) after plugging in for w_i. Therefore, $w_i = 1/2$, implying $p_i^2(o) = 1/2$ and $p_i^1 + \delta p_i^2(i) = (1+\delta)/2$. By positive selection, we get $p_i^2(i) = w_i = 1/2$ and hence $p_i^1 = 1/2$.[23] Prices are constant and all types buy their higher-valued variety in $t = 1$.

Finally, consider (iii) mixed varieties. Note that in period 2, there is no difference between the varieties: every decision is final. The supplier's profit for product a takes the same form as in (ii); the supplier's profit for product b takes the same form as in (i). Thus, we obtain $w_i = 1/2$ as the profit maximizing cut-off values. Prices for variety a are given by (ii) and prices for variety b are given by (i). All buyer types buy their higher-valued variety in $t = 1$.

B.2 Example 2: Pricing dynamics

Let $V = \{v \in [0,1]^2 | v_b = v_a\}$, as illustrated in Figure 5 and $\mathcal{F}(E) = \int_E (1/\sqrt{2})d\mu(v)$, for $E \in \mathcal{B}(V)$, where μ is the Lebesgue measure.

Let us start with the static monopoly prices. Since a and b are perfect substitutes, only the price $p_l = \min\{p_a, p_b\}$ matters, and the demand function $d(p) =$

[22]buyer type $v = (v_a, v_b)$ buys in the first period iff $v_i + \delta v_i - p_i^1 - \delta p_i^2(i) \geq \max\{\delta(v_i - p_i^2(o)), v_j + \delta v_j - p_j^1 - \delta p_j^2(j), \delta(v_j - p_j^2(o)), 0, v_j - p_j^1 + \delta v_i - \delta p_i^2(j), v_i - p_i^1 + \delta v_j - \delta p_j^2(i)\}$.

[23]The price for switchers or non-switchers $p_i(j)$ is set such that the cut-off type is indeed w_i.

$1-p_l$ results. Maximizing the profit $\pi(p) = (1-p_l)p_l$ yields the the monopoly price $p_l^m = 1/2$. Therefore, $p^m \in \{p \in [\psi, 1]^2 | \min\{p_a, p_b\} = 1/2\}$ is the set of (outcome-equivalent) price profiles that maximize static profit.

Next, consider the set of price profiles that leave no trading-up opportunities in the static game. (i) for two durable varieties, $\Omega = \{p \in [\psi, 1]^2 | \min\{p_a, p_b\} \leq 0\}$; (ii) for two rental varieties, $\Omega = \{p \in [\psi, 1]^2 | p_a = p_b \leq 0\}$; (iii) and for mixed varieties, where a is the rental and b is the durable, $\Omega = \{p \in [\psi, 1]^2 | p_b \leq \min\{0, p_a\}\}$. We obtain in all three cases $p^m \notin \Omega$. Therefore, there exist trading-up opportunities at any static monopoly price profile.

Next, we derive a PBE for $T = 2$. Let us start with (i) two durables. Suppose only buyer types $v_i \leq w, i \in \{a, b\}$ remain in the market in period 2.[24] The demand function in period 2 is $(w - p_l^2)$, where $p_l^2 = \min\{p_a^2, p_b^2\}$. Maximizing the profit in the second period results in $p_l^2 = w/2$. Next, let $w = p_l^1 - \delta p_l^2$ be the cut-off type who strategically delays his purchase.[25] The discounted sum of profits is $(1-w)p_l^1 + \delta(w - p_l^2)p_l^2$. Note that the cut-off types determine all prices, thus, we can rewrite the supplier's function in terms of the cut-off value w, which determines when to trade-up which types:

$$(1-w)\left(1 + \frac{\delta}{2}\right) w + \delta \left(w - \frac{w}{2}\right) \frac{w}{2}.$$

The resulting profit maximizing cut-off type is $w = (2+\delta)/(4+\delta)$, implying $p_i^2 = (2+\delta)/(8+2\delta)$ and $p_i^1 = (2+\delta)^2/(8+2\delta)$. Thus, pricing dynamics emerge from trading-up opportunities.

Next, consider (ii) two rental varieties. From above we immediately get the price for first-time renters in period 2, $p_l^2(o) = w/2$. The profit that the supplier obtains from those types is thus equal to the profit that the supplier obtains in (i) from buyer types in the second period. The cut-off type is $w = p_l^1 + \delta p_l^2(l) -$

[24]buyer type $v = (v_a, v_b)$ buys variety i in period 1 iff $v_i + \delta v_i - p_i^1 \geq \max\{\delta(v_i - p_i^2), v_j + \delta v_j - p_j^1, \delta(v_i - p_i^2), 0)\}$. In the equilibrium, which we are constructing, this condition simplifies to $v_i \geq w$.

[25]Because a and b are perfect substitutes, the cut-off type is defined as being indifferent between buying a variety today or tomorrow.

$\delta p_l^2(o)$,[26] where $p_l^2(l)$ is the minimal price in period 2, after buyer types have bought the variety with the minimal price in period 1. The net present value of the price for repeat renters is $p_l^1 + \delta p_l^2(l)$. The supplier, thus, gets the profit $(1 - w)(p_l^1 + \delta p_l^2(l)) + \delta(w - p_l^2)p_l^2$. Plugging in for w, we obtain the equivalent problem as in (i): the supplier has to decide when and which types to trade-up. Therefore, $w = (2+\delta)/(4+\delta)$, implying $p_l^2(o) = (2+\delta)/(8+2\delta)$ and $p_l^1 + \delta p_l^2(l) = (2+\delta)^2/(8+2\delta)$. By positive selection, we get $p_l^2(l) = w = (2+\delta)/(4+\delta)$ and hence $p_l^1 = p_l^2(o)(2-\delta) = (4-\delta^2)/(8+2\delta)$. Comparing the price levels, we obtain that the seller sets a price in period 1 below the static monopoly price, yet if the buyer proceeds with the purchase, the price in the second period is above the static monopoly price; if the buyer does not buy, the relevant price falls. I.e. price dynamics emerge.

Finally, consider (iii) mixed varieties. Note that varieties are perfect substitutes. Thus, the supplier can either price the rental variety a as in (ii), and prices for the durable variety b weakly higher as in (i); or, price the durable variety b as in (i), and price the rental variety a weakly higher as in (ii). Either way, the outcome is equivalent as above: pricing dynamics emerge due to trading-up opportunities.

[26] buyer type $v = (v_a, v_b)$ buys in the first period iff $v_i + \delta v_i - p_i^1 - \delta p_i^2(i) \geq \max\{\delta(v_i - p_i^2(o)), v_j + \delta v_j - p_j^1 - \delta p_j^2(j), \delta(v_j - p_j^2(o)), 0, v_j - p_j^1 + \delta v_i - \delta p_i^2(j), v_i - p_i^1 + \delta v_j - \delta p_j^2(i)\}$. In the symmetric equilibrium, we are constructing, this condition simplifies to $v_i \geq w$.